1983

A rational expectations approach to the modelling of agricultural supply: a case study of Iowa

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A RATIONAL EXPECTATIONS APPROACH TO THE MODELLING OF AGRICULTURAL SUPPLY: A CASE STUDY OF IOWA

Iowa State University

PH.D. 1983

University Microfilms International 300 N. Zeeb Road, Ann Arbor, MI 48106

A rational expectations approach to the modelling of agricultural supply: A case study of Iowa

by

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A Dissertation Submitted to the
Graduate Faculty in Partial Fulfillment of the
Requirements for the Degree of
DOCTOR OF PHILOSOPHY

Major: Economics

Approved:

Signature was redacted for privacy.

In Charge of Major Work

Signature was redacted for privacy.

For the Major Department

Signature was redacted for privacy.

For the Graduate College

Iowa State University
Ames, Iowa

1983

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DEDICATION

This work is dedicated to my late brother Begashaw G. Christos.

CHAPTER I. INTRODUCTION

The issues concerning agricultural production and food supply have concerned economists for years. These concerns have resulted in extensive research in the area of farmers' responses to price changes as typified by the works of Ezekiel (1938), Heady and Kaldor (1954), Nerlove (1956, 1958, 1972, 1979), Behrman (1968), Askari and Cummings (1976) and Nerlove et al. (1979), among others. The central theme of these studies is the quantitative and qualitative understanding of the determinants of the dynamics of agricultural supply and its responses to altered incentives.

Nerlove wrote:

> Whether such market forces, however, impinge directly and visibly on individual farm entrepreneurs, it will nonetheless be true, if we accept the presupposition of optimizing behavior, that shadow prices and opportunity costs are crucial determinants of agricultural supply. It follows that responses to changing "prices" for outputs and inputs, whether made visible by markets, must be a key element in our attempt to understand the agricultural production and food supply . . . (Nerlove, 1979, p. 874).

Different theoretical and empirical methods for evaluating farmer's responses to price changes have been suggested in the literature. One basic difference among these approaches is the assumption made about price expectation formation.

Some Theories of Expectation Formations

As economists have increasingly recognized the importance of expectations in determining economic behavior, they have attempted to incorporate within their behavioral models some representation of the mechanism by which economic agents form their expectations. How do agents form their expectations about future outcomes of economic variables? What kind of information is used? How are different pieces of information combined together to make predictions about the future? Attempts to answer these questions have generated considerable discussion and debate and several by potheses. It should be noted, however, that although data on agents anticipation are collected in many ways, only a few studies have tried to determine how individual decision-makers actually form expectations (Heady and Kaldor, 1954; Turnovsky, 1970; Fisher and Tanner, 1978; and Nerlove, 1983).

The most popular device for representing expectations formation has been distributed lags on the variable in question. The general form is

$$ {}_{t-1}X_t^e = \sum_{i=0}^{\infty} W_i X_{t-1-i} \qquad (1.1) $$

where ${}_{t-1}X_t^e$ is the expectation of X at time t held at time t-1 (hereafter written as X_t^e); X_t is observed X at time t. The underlying assumption is that economic agents form forecasts about future values of X based entirely on its past history. Two important issues are determination of W's and lag length.

Most of the popular expectations formation rules used in economic studies are special cases of Equation (1.1).

1. Static expectations: This is the simplest of all expectation theories. It states that the forecast of a variable, say price, for the period t+1 is the currently observed price, i.e.,

$$P^e_{t+1} = P_t \tag{1.2}$$

It is in the general class of (1.1) where $W_o=1$ and $W_i=0$, $i\neq 0$. The theory is based on the assumption of no memory and no learning by economic agents.

2. Extrapolative expectations: Under extrapolative expectations theory, the expected value of a variable, say price, is defined as

$$P^e_{t+1} = P_t + \eta(P_t - P_{t-1}) \tag{1.3}$$

where P^e_{t+1} and P_t are as defined above and η is a coefficient of expectation. The purpose of the extrapolative expectation is to modify the static expectation theory to take into account the most recent trend in prices. If $\eta=0$, this model is identical to static expectations. If $n>0$, the expected price will be the weighted sum of the present and the past prices with weights $(1+\eta)$ and $-\eta$ for P_t and

P_{t-1}, respectively.

This model assumes a simple learning process on the part of the economic agent such that the expected price for next period is the actual price for the present period plus (or minus) some proportion of the change in the actual price between one period ago and the present period. This approach, while more satisfactory than the simple static theory, is nonetheless rather naive itself. Economic agents are still assumed to have very short memories.

3. Adaptive expectations: During the 1960s and early 1970s, this theory became very popular. According to this theory, individuals are assumed to revise their expectations according to the most recent experience:

$$P^e_{t+1} = P^e_t + \theta(P_t - P^e_t) \qquad (1.4)$$

where θ is the coefficient of expectations and the other variables are as defined earlier. The purpose of the adaptative expectation theory is to permit agents to adjust expectation to take account of immediate past errors in expectation formation. Rearranging (1.4), we obtain

$$P^e_{t+1} - (1-\theta)P^e_t = \theta P_t \qquad (1.5)$$

Replacing $(1-\theta)$ by β, we obtain

$$[(1-\beta)L]P^e_{t+1} = (1-\beta)P_t \qquad (1.6)$$

where L is the lag operator such that $L^j X_t = X_{t-j}$; so that

$$P^e_{t+1} = \left(\frac{(1-\beta)}{1-\beta L}\right) P_t \quad (1.7)$$

Provided $|\beta| < 1$, we can expand $\frac{1}{1-\beta L}$ as $1 + \beta L + \beta^2 L^2 + \beta^3 L^3 + \ldots$ and thus write (1.7) as

$$P^e_{t+1} = (1-\beta) \sum_{K=0}^{\infty} \beta^K P_{t-k} \quad (1.8)$$

Under the adaptive expectation hypothesis, the expected price may be expressed as an infinite weighted average of past observed prices with weights declining geometrically as the lag length increases.

Econometricians have a long history of using distributed lags to represent expectations. If the same process generates future as past outcome, the distributed lagged models, using past values of a variable, give the best forecast of the variable. In general, these approaches to expectations perform well for the sample period; however, their performance for forecasting beyond the sample period is questionable. If the structure of the economy changes, there is no mechanism in distributed lag models to capture these changes.

4. Rational expectations: In his paper, "Rational Expectations and the Theory of Price Movements", Muth (1961) develops a rational expectations model that eliminates the theoretical weakness common to previous theories of expectation formation. Muth's theory is based on three hypotheses about individual behavior.

> (1) Information is scarce, and the economic system generally does not waste it. (2) The way expectations are formed depends specifically on the structure of the relevant system describing the economy. (3) A "public prediction", in the sense of Grunberg and Modigliani, will have no substantial effect on the operation of the economic system (unless it is based on inside information) (Muth, 1961, p. 316).

This theory implies that economic behavior underlies the formation of expectations and that expectations are based on information, which is assumed implicitly to be costless. Rational expectations, by Muth's definition, states that economic agents form their expectations as if they know the process which will ultimately generate the actual outcomes in question; i.e., agents subjective probability distribution describing the future outcomes are identical to the corresponding objective probability distribution conditional on the "true" model of the economy.

Although Muth's definition of rational expectations seems to be straight forward, there are other definitions of "rational expectations" in the economics literature (see, for example, Friedman, 1979). Rawls noted that "one might reply that the rationality of a person's choice does not depend upon how much he knows, but only upon how well he reasons from whatever information he has, however incomplete. Our decision is perfectly rational provided that we face up to our circumstances and do the best we can" (Rawls, 1971).

Much of the confusion surrounding the concept of rational expectations stems from the failure to distinguish between (a) the general assumption that economic agents use efficiently, given available information; and (b) a specific assumption identifying the available information. There is a general agreement on (a). The specific information availability assumption (b) in Muth's rational expectations hypothesis (REH) is that the information which is available to economic agents is sufficient to permit them to form expectations characterized by conditional subjective distribution of outcomes that are equivalent to the conditional objective distribution of outcomes indicated by the "relevant economic theory".

If an economic agent's expectation about a future value of a variable is the same as the predictions of the relevant economic theory, then his expectation is rational (in the sense of Muth). Muth's theory does not say how economic agents derive the knowledge which they use to formulate expectations meeting these requirements. As noted by Wallis (1980), "the informational requirement of rational expectations has led some to doubt the empirical applicability of these models but this seems to be as yet unresolved . . .".

In this dissertation, Muth's concept of rational expectations is adopted. In order to derive the price expected to prevail at $(t+1)^{th}$ period on the basis of information through t^{th} period, P^e_{t+1}, Muth assumes ". . . (1) The random disturbances are normally distributed; (2) certainty equivalence exists for the variable to be predicted; and (3) the equations of the system, including the expectation formulas, are linear". Mathematically, the price expected to prevail at time t+1 is equal to the conditional mathematical expectation of price, which is the mathematical expectation of price conditional on information available through time t, or

$$E_t(P_{t+1}/\Omega_t) = P^e_{t+1} \tag{1.9}$$

where

E_t is expectation operator,

Ω_t is information set available at time t.

Muth's rational expectations framework requires economic agents to have a structural model and utilize all available information. An agent's expectation about future price outcome changes if new information becomes available. The information includes the laws of motion that describe the exogenous stochastic variables such as prices of outputs and inputs, innovations, government policy variables and other relevant variables. A change in the economic agent's perception of the laws of motion that govern these variables will change the decision rule for choice variables.

Consider the following example adopted from Muth's 1961 paper:

$$D_t = -\beta P_t, \ \beta>0 \ \text{(Demand)}$$

$$S_t = \gamma(P_t^e) + u_t, \ \gamma>0 \ \text{(Supply)} \tag{1.10}$$

$$D_t = S_t \ \text{(Market equilibrium)}$$

where D_t is the amount consumed, S_t represents the number of units produced in a period lasting as long as the production lag, P_t is the market price in the t^{th} period and P_t^e is the market price expected to prevail at t^{th} period on the basis of information available through the $(t-1)^{th}$ period. u_t is stochastic disturbance, e.g., variation in yield due to weather. All variables are in terms of deviations from equilibrium values. Solving (1.10) for P_t, we obtain

$$P_t = -\frac{\gamma}{\beta}(P_t^e) - \frac{1}{\beta}u_t \tag{1.11}$$

If there is no serial correlation of u_t, and $E_{t-1}(u_t) = 0$, then we obtain

$$E_{t-1}(P_t) = -\frac{\gamma}{\beta}(P_t^e) \tag{1.12}$$

By the rational expectations assumption (1.9), $P_t^e = 0$ or P_t^e is equal to the equilibrium price.

Now, suppose there is serial correlation among the u's and that they can be written as a linear combination of the past history of normally and independently distributed random variables, e_t, with zero mean and variance σ^2:

$$u_t = \sum_{i=0}^{\infty} W_i e_{t-i}$$

$$E(e_j) = 0 \qquad (1.13)$$

$$E(e_i e_j) = \begin{cases} \sigma^2, & \text{if } i=j \\ 0, & \text{if } i\neq j \end{cases}$$

$$u_t^e = E(u_t/\ldots e_{t-2} e_{t-1}) = \sum_{i=1}^{\infty} W_i e_{t-i} \qquad (1.14)$$

Taking expected value of (1.11) and substituting (1.14), we obtain

$$P_t^e = -\frac{1}{\beta+\gamma} \sum_{i=1}^{\infty} W_i e_{t-i} \qquad (1.15)$$

From (1.11),

$$P_t = -\frac{\gamma}{\beta} P_t^e - \frac{1}{\beta} u_t$$

$$= \frac{\gamma}{\beta}\left(\frac{1}{\gamma+\beta}\right) \sum_{i=1}^{\infty} W_i e_{t-i} - \frac{1}{\beta} \sum_{i=0}^{\infty} W_i e_{t-i}$$

$$= -\frac{W_0}{\beta} e_0 - \frac{1}{\gamma+\beta} \sum_{i=1}^{\infty} W_i e_{t-i} \qquad (1.16)$$

or

$$P_t = \sum_{i=0}^{\infty} \Pi_i e_{t-i} \qquad (1.17)$$

and

$$P_t^e = \sum_{i=1}^{\infty} \Pi_i e_{t-i} \qquad (1.18)$$

where

$$\Pi_0 = -\frac{W_0}{\beta}$$

$$\Pi_i = -\frac{W_i}{\gamma+\beta}, \quad i>0.$$

The e's are not observable. Therefore, we need to write P_t^e in terms of the past history of prices, i.e.,

$$P_t^e = \sum_{j=1}^{\infty} V_j P_{t-j} \quad . \tag{1.19}$$

We can solve for the V's in terms of the Π's in the following manner. From (1.19) and (1.18)

$$\sum_{i=1}^{\infty} \Pi_i e_{t-i} = \sum_{j=1}^{\infty} V_j P_{t-j} \quad . \tag{1.20}$$

Substituting (1.17) into (1.20), we obtain

$$\sum_{i=1}^{\infty} \Pi_i e_{t-i} = \sum_{j=1}^{\infty} V_j \sum_{i=0}^{\infty} \Pi_i e_{t-i-j}$$

$$= \sum_{i=1}^{\infty} \left(\sum_{j=1}^{i} V_j \Pi_{i-j} \right) e_{t-i} \quad . \tag{1.21}$$

Since the equality must hold for all e's, the coefficient must satisfy the equations

$$\Pi_i = \sum_{j=1}^{i} V_j \Pi_{i-j}, \quad i = 1,2,3,... \tag{1.22}$$

The V's can successively be solved from the Π's which are themselves functions of β, γ and w's. If we assume that $W_i = 1$ for all i = 1,2,..., then

$$\Pi_0 = - \frac{1}{\beta}$$

$$\Pi_i = - \frac{1}{\gamma+\beta} \tag{1.23}$$

From (1.22) it can be seen that the expected price is a geometrically weighted moving average of past prices:

$$P_t^e = \frac{\beta}{\gamma} \sum_{j=1}^{\infty} \left(\frac{\gamma}{\beta+\gamma}\right)^j P_{t-j} . \qquad (1.24)$$

It is important to note that the result that expected price is a weighted average of past prices, (1.24), followed directly from the fact that P_t could be expressed as a weighted sum of the e's alone. If there are other exogenous variables in the model (1.10), then the rational expectations of price will involve the past history of those exogenous variables as well as P_t (Nelson, 1975b).

It is helpful at this point to summarize the four expectational regimes in Table 1.1.

Table 1.1. Comparative expectional regimes

Expectations	P_{t+1}^e	
Static	P_t	
Extrapolative	$P_t + \eta(P_t - P_{t-1})$	
Adaptive	$(1-\beta) \sum_{k=0}^{\infty} \beta^k P_{t-k}$,	$\lvert\beta\rvert<1$
Rational	$\frac{\beta}{\gamma} \sum_{j=0}^{\infty} \left(\frac{\gamma}{\gamma+\beta}\right)^j P_{t-j}$,	$\beta>0$, $\gamma>0$

In the first three expectional regimes, the coefficients of the distributed lags are not derived from structural models. In the fourth regime, the coefficients are a non-linear function of the parameters (β and γ) of the underlying model. Thus, with rational expectations, any change in the structural parameters β and γ will change P_{t+1}. Such changes do not affect P^e_{t+1} under the other expectional regimes.

The REH implies that agents know the structure of the system in which they operate and form their expectations about future variables within the framework of the system. A change in the structure will induce economic agents to revise their expectations accordingly. To use Sim's (1980) example, it is possible that upon reading news of a frost in Brazil, U.S. consumers will stockpile coffee in anticipation of a price increase. The implication is that variables known to affect coffee supply also enter the coffee demand equation (and vice versa) through their effect on expected prices. Therefore, the optimal price predictor will contain information from both the supply and the demand equations. This implication agrees with the evidence presented by Heady and Kaldor on farmers; formation of price expectations.

> For their 1948 and 1949 forecasts, the majority was not using simple mechanical models such as the projection of the current price or recent price trend in the next year but was attempting to analyze and predict the more complex price making forces. A rather common procedure appeared to start the process of devising expected prices from current prices.

> The current price then was adjusted for the expected effect of important supply-and-demand forces. Where farmers possessed little information about these forces, there was a tendency to project either the current price or the recent price trend (Heady and Kaldor, 1954).

Price Expectations and Agricultural Supply

The general practice in agricultural supply models has been such that when expected future values of a variable were thought to be important in a behavioral equation, they were replaced by a distributed lag on that same variable. Early studies (e.g., Ezekiel, 1938), were based on the assumption of static expectations; i.e., prices observed at the time of planting were expected to prevail at the time of harvest. Using static expectations, researchers were able to explain the observed oscillatory movements in some agricultural output and prices (Cobweb models). The computed elasticities implied that farmers were not responsive to price changes, but this conclusion was contradicted by farmers' behavior under the price support system (Nerlove, 1958; Cochrane and Ryan, 1976).

Nerlove's early work (1956, 1958) made a landmark in the area of supply response functions. He showed that insufficient attention had been given to the problem of identifying the price variable to which farmers react. This was the principal reason other researchers obtained small estimates of price elasticities of crop supply.

Using adaptive expectations, Nerlove (1956, 1958) showed that farmers' responses to changes in prices can be represented by a distributed lag. In particular, he derived an equation for current area (a proxy for output) as a function of lagged area, lagged prices and other current and lagged exogenous variables. The coefficients of the model are nonlinear functions of the parameters of a linear supply function, an adjustment parameter between desired and actual acreage, and an adaptive expectation parameter.

More recently, the adequacy of adaptive expectations as a representation of agents' forecasts of future variables has been criticized (Sargent and Wallace, 1976; Nerlove, 1979; Wallis, 1980; and Goodwin and Sheffrin, 1982). Adaptive expectation is not criticized because it implies that expected price is some weighted average of present and all past prices. It is, however, criticized because it requires some ad hoc assumptions about the parameters of the lag process and it does not consider other relevant variables. In particular, in the Nerlove-type agricultural supply analysis, the model's parameters are implicitly assumed to be independent of the process that generates crop prices. Therefore, the estimated coefficients are invariant to changes in government policies which influence the paths of the price process. Thus, these models are subject to Lucas's criticism of economic policy evaluations (Lucas, 1981a). Here, the emphasis is on a change in policy rules or regimes, rather than a change in a particular value of a policy instrument.

An alternative to adaptive expectations is offered by the rational expectations hypothesis of Muth (1961). With rational expectations, agents are assumed to take account of the interrelationships among economic variables. In particular, the hypothesis, as presented above, states that "expectations, since they are informed predictions of future events, are essentially the same as the predictions of the relevant economic theory" and hence, depend "specifically on the structure of the relevant system describing the economy" (Muth, 1961). If we can accept that farmers are rational in the sense that they are optimizers, then rational expectations can provide a framework to circumvent problems associated with adaptive expectations.

An additional shortcoming of the Nerlovian models is that the dynamic element in the basic supply response model is introduced without a formal theory. The simple ad hoc assumption is that each period, if we are dealing with discrete time, a fraction of the difference between the current position and the long-run equilibrium is eliminated (Nerlove, 1979). There is, however, no need to follow an ad hoc strategy because the production process provides most of the essential dynamic structure. The current yield (productivity) of land depends crucially on how land was employed during previous periods. The deterioration of land productivity under some cropping patterns introduces a nontrivial dynamic element in the allocation of land between crops (see, for e.g., Eckstein, 1981).

For a given technology, producing some crops cause the land or soils to deteriorate faster than producing others. This deterioration occurs in the form of soil erosion, depletion of some essential nutrients for plant growth and build-up of harmful pest population. Compare growing corn and soybean. Soybean causes a significantly higher rate of soil erosion because the fibrous roots of the plant extensively loosen the soils. Soybeans also have a beneficial effect of increasing the nitrate content of soils. Corn, on the other hand, is less erosive than soybeans, but it depletes the nitrate content of soils. To maintain the soil fertility, farmers use fertilizes and/or practice crop rotation. Cyclical movement in corn and soybean output can be attributed to farmers' choice of technology (crop rotation) rather than to their naive price expectations formation process (static expectations).

Furthermore, crop production is subject to some forces beyond farmers' control, e.g., weather and natural soil characteristics. The price level and shocks to agricultural productivity can be viewed as uncontrollable stochastic processes that affect farmers' net incomes. Hence, farmers' choices of tillage practices, fertilizer and pesticide application rates, and cropping systems can be represented as outcomes of a stochastic dynamic optimization problem that they solve.

Objective of the Study

In recent years, there has been increasing interest in the application and econometric implication of the REH. A considerable amount of the applied work so far has been in macromodels; especially in the area of monetary economics. A number of papers, such as Lucas (1981b), Nelson (1975a,b), McCallum (1976a,b), Blanchard and Kahn (1980), Wallis (1980), Taylor (1979), Hansen and Sargent (1981a,b) have discussed the estimation of models which contain rational expectations. Another group of papers, including Revankar (1980), Wallis (1980) and Hoffman and Schmidt (1981) have discussed testing the restrictions implied by REH. Actual models embodying the REH have been estimated by Sargent (1976, 1978a,b, 1979), Taylor (1979); among the others. Stanley Fischer (1980) edited a number of papers concerned with the rational expectations and economic policy issues. However, few studies have proposed a rational expectation's version of agricultural supply (Huntzinger, 1979; Eckstein, 1981; Goodwin and Sheffrin, 1982; and Fisher, 1982). Even fewer studies have attempted to estimate models and to test restrictions imposed by the REH within the context of agricultural supply.

The objective of this research is then to build a dynamic model of agricultural supply where expectations of exogenous variables are assumed to be formed rationally and to test the restrictions implied by the REH. Specifically, farmers' choices of outputs and inputs are derived from a model of optimizing behavior. Farmers are assumed to make choices that maximize the expected present value of their income stream subject to dynamic and stochastic technology and their information. The dynamics arise from the technology; and, the assumption that farmers form their expectations rationally implies that they know the actual distribution generating the exogenous variables. Hence, farmers' decision rules depend on the parameters of the actual dynamic process of prices, including government as policies.

The theme of this work is that land allocation and outputs supplied are outcomes of an optimizing process. By specifying an explicit approximation of the optimization problem that farmers are assumed to solve, we hope to improve our qualitative and quantitative understanding of farmers' decision-making process and behavior.

The model gives rise to a system of simultaneous equations containing equations for acreage, crop yield, price ratios, and other exogenous variables. Estimates of parameters of the model are obtained by fitting the system of equations to aggregate time series data. The worth of this model is of course dependent on how well the model explains the data. The dynamic rational specification of the model gives a set of testable restrictions on parameters. One test of "the" model is to see if these restrictions implied by the theory are supported by the data. The model will be fitted to Iowa aggregate time series data on soybeans, corn, and other related variables for the period 1948-80.

Organization of Report

In the first chapter, some important issues concerning the formation of expectations have been reviewed. In particular, recent developments in the theory of rational expectations can remedy some of the shortcomings of traditional specifications of agricultural supply functions. The objective of this dissertation is to develop a dynamic rational expectations model of agricultural supply.

The rest of the report is organized as follows. Chapter

II develops the theoretical framework of the model. In Chapter III, a vector time series model is utilized to perform some preliminary tests. Some of the assumptions underlying the model will also be tested in this chapter. Chapter IV includes some more detailed discussion of the data and the empirical results, while Chapter V contains summary and conclusions as well as some conjecture for future research.

CHAPTER II. THEORETICAL FRAMEWORK AND MODEL SPECIFICATION

This chapter emphasizes theoretical aspects of models containing rational expectations. Procedures for formulating and estimating rational expectation models are discussed. In the context of agricultural supply, the rational expectations model derived in this chapter is observationally equivalent to the usual agricultural supply functions. However, the rational expectation model is optimal in each time period and the parameters of the model have different interpretations from the supply functions.

In the first part of the chapter, two approaches to the formulation and estimation of rational expectation models are discussed. A model of land allocation under rational expectations is presented in the second part.

Rational Expectation Models

There exists in the recent literature, two common methods for incorporating the REH into econometric models. In the first method, an economic agent is assumed to maximize a constrained objective function. In particular, the agent is assumed to maximize his expected income stream subject to some technological constraints. The maximization problem can be formulated in an infinite or finite time horizon. Then, by imposing rational expectations by postulating some autoregressive moving average (ARMA) processes for all the nonchoice variables in the model, a closed-form solution to the problem can be obtained. To make the control problem mathematically tractable, the return function is assumed to be linear-quadratic. The solution to the maximization problem is a set of stochastic processes, some of which describe the agents' decision rules. As a result of REH, within-equation and cross-equation parameter restrictions are imposed on the equations describing the decision rules and equations describing the laws of motion for the other exogenous variables.

The strategy for estimating the above model-types is to jointly estimate the equations for agents' decision rules and the equations for the stochastic processes describing exogenous variables, subject to within and cross restrictions implied by the REH. However, even for very simple models, the cross-equation restrictions are of complicated form because they contain nonlinear restrictions on the parameters of the model. The formulation and estimation of such models is discussed in Hansen and Sargent (1981a,b).

The above approach provides a tractable procedure for combining econometric methods and dynamic economic models for the purpose of modeling and interpreting economic time series. Thus, the estimation strategy has the advantage of combining time series analysis and traditional econometric estimation techniques. The time series analysis is utilized to generate the necessary forecasts of the exogenous variables. The second method, also follows an integrated time series - econometric approach.

In the second method, REH is imposed on a traditional simultaneous econometric model that contains expected values of the endogenous variables (or a subset). In this approach, the equations in the system describe both the optimal decision rules of the agents and the way they interact with each other. See Wallis (1980) and McCallum (1976a).

As an example of the second approach, we present a generalization of Wallis' (1980) model:

$$By_t + Ay_t^e + \Gamma_1 X_{1t} + \Gamma_2 X_{2t} = U_t \qquad (2.1)$$

where y_t is a vector of g endogenous variables, y_t^e is a vector of g anticipated values of the endogenous variables formed in period (t-1), X_{1t} is a K_1 vector of uncertain exogenous variables, X_{2t} is a $(K-K_1)$ vector of intercept and seasonal terms whose future values are known with certainty. For simplicity, assume there are no lagged endogenous variables in the system. The parameter matrices B, A, Γ_1 and Γ_2 have dimensions (gxg), (gxg), (gxK_1) and $[gx(K-K_1)]$, respectively.

The expected variables, y_t^e, are unobservable and it is assumed that expectations are formed rationally, i.e., $y_t^e = E(y_t|\Omega_{t-1})$. From (2.1) note that

$$By_t + Ay_t^e = U_t - \Gamma_1 X_{1t} - \Gamma_2 X_{2t} \quad (2.2)$$

Taking conditional expectations, we obtain

$$(B+A)y_t^e = -\Gamma_1 \hat{X}_{1t} - \Gamma_2 X_{2t} \quad (2.3)$$

where

$$\hat{X}_{1t} = E(X_{1t}|\Omega_{t-1}).$$

From (2.3) it follows that

$$y_t^e = -(B+A)^{-1}\Gamma_1 \hat{X}_{1t} - (B+A)^{-1}\Gamma_2 X_{2t} \quad (2.4)$$

Thus, rational expectations, y_t^e, are a linear combination of the predicted values of uncertain exogenous variables ($\hat{X}_{1t}$) and of actual values of certain exogenous variables (X_{2t}). In order to complete the specification of the model, we need to specify the process by which the vector of uncertain exogenous variables, X_{1t}, is generated. This is usually done by postulating a vector ARMA process for X_{1t} or univariate ARMA process for each component of X_{1t}.

To write (2.1) in terms of observable variables, substitute (2.4) into (2.1) to obtain

$$By_t - A(B+A)^{-1}\Gamma_1 \hat{X}_{1t} + \Gamma_1 X_{1t} - A(B+A)^{-1}\Gamma_2 X_{2t} \quad (2.5)$$

$$+ \Gamma_2 X_{27} = U_t$$

The reduced form of the system then becomes

$$y_t = B^{-1}A(B+A)^{-1}\Gamma_1\hat{X}_{1t} - B^{-1}\Gamma_1 X_{1t} + B^{-1}A(B+A)^{-1}\Gamma_2 X_{2t}$$

$$+ B^{-1}U_t \ . \tag{2.6}$$

The observed value of y_t is determined by predicted and actual values of uncertain exogenous variables and actual values of certain exogenous variables.

Procedures for identifying and fitting this type of models are discussed in Wallis (1980), Chow (1980), McCallum (1976a).

The error in rational expectations is the difference between y_t and y_t^e, i.e.,

$$y_t - y_t^e = -B^{-1}\Gamma_1(\hat{X}_{1t}-X_{1t}) + B^{-1}U_t \tag{2.7}$$

The error depends solely on the unanticipated part of the current exogenous variables and current disturbances. Wallis (1980) and Nelson (1975a) have shown that $E[y_t-y_t^e]^2$ is smaller than for any other (nonrational) expectional rules. Therefore, rational expectation has an optimal property of smallest mean-squared-error among expectational formulas. Nelson (1975a) has also shown that RE are efficient in the broader sense of maximizing expected utility for each market participant.

To illustrate policy analysis via rational expectations, consider a specific case of (2.1).[1]

$$Q_t + \alpha_{11}P^e_t + \alpha_{12}(PI_t - S_t) + \alpha_{13} = v_{1t} \text{ (supply)}$$

$$\alpha_{21}Q_t + P_t + \alpha_{22}z_t + \alpha_{23} = v_{2t} \text{ (Demand)} \qquad (2.8)$$

The quantity supplied Q_t, is a function of the price expected in period t, P^e_t, the price of inputs, PI_t, and the value of an input subsidy (or tax), S_t. In this case, S_t may represent a subsidy paid on fertilizer, or it could represent an excise tax on fuel prices. Price, P_t, is specified as a function of the quantity sold and disposable income, z_t. Expectations are formed rationally, and the market is assumed to clear each period. Define $y_t = (Q_t\ P_t)'$

$$X_{1t} = (PI_t S_t Z_t)'$$

$$U_t = (v_{1t}\ v_{2t})$$

and apply the procedures outlined in Equations (2.2)-(2.6), then the reduced form for (2.8) is:

$$Q_t = \mu_{11}\hat{PI}_t + \mu_{12}\hat{S}_t + \mu_{13}\hat{Z}_t + \mu_{14}PI_t + \mu_{15}S_t + \mu_{16}Z_t + \mu_{10} + v_{1t} \qquad (2.9)$$

$$P_t = \mu_{21}\hat{PI}_t + \mu_{22}\hat{S}_t + \mu_{23}\hat{Z}_t + \mu_{24}PI_t + \mu_{25}S_t + \mu_{26}Z_t + \mu_{20} + v_{2t}$$

[1]This example is adopted from Fisher (1982).

where μ_{ij} is some nonlinear function of α_{11}, α_{12}, α_{21}, α_{13}, α_{23} and α_{22}; $\hat{X}_t$ denotes the forecasts of X_t.

Equation (2.9) can be employed to analyze the impact on output of a change in the subsidy. Assume the stochastic processes underlying S_t, PI_t and Z_t are

$$\begin{aligned} S_t &= \phi_1 S_{t-1} + e_{1t} \\ PI_t &= \phi_2 PI_{t-1} + e_{2t} \\ Z_t &= \phi_3 Z_{t-1} + e_{3t} \end{aligned} \qquad (2.10)$$

where e_{it} is a white noise process independent of U_t; so that

$$\begin{aligned} \hat{S}_t &= \hat{\phi}_1 S_{t-1} \\ \hat{PI} &= \hat{\phi}_2 PI_{t-1} \\ \hat{Z}_t &= \hat{\phi}_3 Z_{t-1} \end{aligned} \qquad (2.11)$$

Then, from Equations (2.8), the quantity equation becomes

$$\begin{aligned} Q_t = \mu_{10} &+ \mu_{11}\phi_2 PI_{t-1} + \mu_{12}\phi_1 S_{t-1} + \mu_{13}\phi_3 Z_{t-1} \\ &+ \mu_{14} PI_t + \mu_{15} S_t + \mu_{16} Z_t + v_{1t}, \end{aligned} \qquad (2.12)$$

Announced and unannounced changes in S_t have different effects on the actual quantity in the market. The impact of an unannounced change in S_t on the quantity produced is given by the coefficient μ_{15}, because $\hat{S}_t$ is unchanged.

Alternatively, the government might announce that it plans to phase out the subsidy evenly over a four-year period. One might calculate the change in Q_t from Equation (2.12) by lowering S_t by 25%. This type of adjustment is labeled as "traditional" approach. However, when the government announces the plan, the process generating S_t changes. This announced policy might be represented by:

$$S_t = S_{t-1} - .25\overline{S}, \text{ for the first four years}$$

$$= 0, \text{ after four years} \qquad (2.13)$$

where $\overline{S}$ is the initial subsidy level. The predictor for S_t is

$$\hat{S}_t = S_{t-1} - .25\overline{S} \qquad (2.14)$$

Substituting (2.14) and the predictors for PI_t and Z_t (Equation 2.11) into the quantity Equation (2.19), we obtain

$$Q_t = \mu_{10} - .25\mu_{12}\overline{S} + \mu_{11}\phi_2 PI_{t-1} + \mu_{12}S_{t-1}$$

$$+ \mu_{13}\phi_3 Z_{t-1} + \mu_{15}S_t + \mu_{16}Z_t + v_{1t} \qquad (2.15)$$

Let us compare Equations (2.15) and (2.12). Some of the reduced form parameters have changed. Equation (2.15) contains new coefficients for S_{t-1} and a new term $-.25\mu_{12}\overline{S}$. It is obvious that Equation 2.12 gives a very different prediction about the change in Q_t due to an announced change in policy than does Equation (2.15). Using Equation 2.12 might be considered the "traditional" econometric practice.[2]

In summary, there are at least two ways in which rational expectations can be incorporated in simultaneous equation models: first, by specifying a constrained objective function where the future value of any variable is taken to be its conditional expectation; second, by imposing rational expectations on some or all of the endogenous variables in the usual system of simultaneous equations. In either case, the REH results in some restrictions being imposed on the models parameters. These restrictions are often called the "hallmark" of rational expectations. In the following section, we follow the first approach to develop an agriculture supply model.

A Land Allocation Model

For almost all crops, yield (productivity) of land depends on how land was employed the previous periods. Production of some crops (e.g., corn) results in a severe soil fertility deterioration due to the nitrate depletion from the soil. On the other hand, production of leguminous plants (e.g., soybean) supplement the nitrate content of the

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[2]Anderson (1979) has devised a method for making rational forecasts from unrational models.

soil. However, soybeans cause a significantly higher rate of soil erosion. Farmers use fertilizers and/or practice crop rotation in order to maintain soil fertility.

Furthermore, farmers are faced with uncontrollable forces such as weather and natural soil characteristics. Thus, farmers choice of input and cropping patterns can be represented as a stochastic dynamic optimization problem. The problem can be a complicated dynamic programming problem and the solution might require dynamic programming procedures. Under a certain set of assumptions, however, the problem can be solved by econometric techniques. In this dissertation, the following simplifying assumptions are made in order to solve the dynamic optimization problem by econometric techniques:

(1) Farmers are risk-neutral so that maximization of expected profit is equivalent to maximization of expected utility.

(2) Relative crop prices are exogenously determined; i.e., the allocation of land between crops and the quantities of outputs supplied by farmer(s) in any one state do not affect relative output prices.

(3) The production is one period long. Decisions on inputs in period t result in output in period t+1.

(4) A representative farmer has a given land and has the option of allocating this land to either crop 1 or crop 2.

This land allocation must be made before the output prices are known.

(5) Finally, the only variable input of the farmer is land.

Farmers are assumed to make choices that maximize the expected present discounted value of their profit subject to some technological constraint. To present the problem in a mathematical form, consider the definition of the following variables.

X_{1t} = output of crop 1 at time t

X_{2t} = output of crop 2 at time t

A_{1t} = land allocation to crop 1 at time t

A_{2t} = land allocation to crop 2 at time t

$\bar{A}_t$ = total cultivated land available at time t

P_{1t} = price of crop 1 at time t

P_{2t} = price of crop 2 at time t

C'_{1t} = cost of production (per acre) of crop 1

C'_{2t} = cost of production (per acre) of crop 2

W_t = a qxl vector containing variables that help predict future variables

a_{1t} = shock to productivity of A_{1t}

a_{2t} = shock to productivity of A_{2t}

E = the mathematical expectation operators,
$E_t(X_{t+1}) = E(X_{t+1}|\Omega_t)$

Ω_t = information set available at time t;
$\Omega_{t-1} \subset \Omega_t$.

β = a discount factor.

The representative farmer's problem is to choose a land allocation plan (A_{1t+j}) to maximize expected present discounted value of farm profits.

$$E_0 \sum_{t=0}^{\infty} \beta^t [\beta P_{1t+1} X_{1t+1} + \beta P_{2t+1} X_{2t+1} - c'_{1t} A_{1t} - c'_{2t} A_{2t}] \qquad (2.16)$$

The maximization is subject to land and technology constraint. The land constraint is:

$$A_{1t} + A_{2t} = \bar{A}_t . \qquad (2.17)$$

The production function for crop 1 is:[3]

$$X_{1t+1} = [d_0 - \frac{d_1}{2} A_{1t} + d_2(\bar{A}_t - A_{1t-1}) + a_{1t}] A_{1t} \qquad (2.18)$$

and the production function for crop 2 is:

$$X_{2t}+1 = [d_3 - \frac{d_4}{2} A_{2t} + d_5(\bar{A}_t - A_{2t-1}) + a_{2t}] A_{2t} \qquad (2.19)$$

where d_0, d_1, d_2, d_3, d_4, and d_5 are production parameters and they all have positive signs, and a_{1t} and a_{2t} are shocks to productivity (yield) in the production of crop 1 and crop 2, respectively.

The terms $d_2(\bar{A}_t - A_{1t-1})$ and $d_5(\bar{A}_t A_{2t-1})$ imply that the yield of each crop in period (t+1) increases proportional to

[3]The production functions are formulated to be quadratic so that (1) they meet the concavity conditions; and (2) following the tradition in such maximization problems, the maximization problem has a linear-quadratic set-up which leads to a tractable expression for the solution. Note that after substituting (2.18) and (2.19) into (2.16), the objective function is quadratic in A_{1t} with a linear constraint (2.17).

the amount of current land which has not been used for the same crop during the previous period (t-1). In other words, the yield of each crop at time (t+1) is assumed to be inversely related to the amount of land allocated to that particular crop at time (t-1) and directly related to the amount of total cultivated land available at time $t(\bar{A}_t)$. If $\bar{A}_t$ is increasing, farmers would have more land which was not planted to neither crop during the previous period which means that they are more flexible and can avoid the loss in yield due to land productivity deterioration.

These specifications imply that the current marginal product of past land allocation for each crop is negative. Some crops (e.g., soybean) have the positive effect of supplementing the nitrate of the soil, and at the same time, they have the negative effect of making the soil erode rapidly. The parameters d_2 and d_5 capture the net effect of past cropping patterns on current production. The hypothesis that $d_2, d_5 > 0$ is that producing the same crop year after year on the same plot of land results in reduced crop yield.

The simple quadratic form of the production functions of crop 1 and crop 2 (strictly concave in A_{1t} and A_{2t}, respectively), enables us to obtain a linear analytical solution to the maximization problem. This linear-quadratic set-up is similar to the linear-quadratic version of Lucas and Prescott's model of investment under uncertainty, Sargent's dynamic labor demand model (Sargent, 1978b), and to the more general class of Hansen and Sargent's dynamic linear rational expectation models (Hansen and Sargent, 1981a).

By substituting (2.17), (2.18) and (2.19) into (2.16), the objective function is restated as a function of one choice variable per time period, A_{1t}

$$\max_{\{A_{1t}\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t \{(d_0+c_{2t}-c_{1t}+a_{1t})A_{1t} - \frac{d_1}{2}A_{1t}^2 + d_2\bar{A}_t A_{1t}$$

$$- d_2 A_{1t-1}A_{1t} + P_{t+1}\bar{A}_t - P_{t+1}A_{1t}\} \qquad (2.20)$$

where

$P_{t+1} = \dfrac{P_{2t+1}}{P_{1t+1}} \cdot \dfrac{X_{2t}}{A_{2t}}$ is the shadow price of crop 1 land allocation;

$c_{1t} = \dfrac{c'_{1t}}{\beta P_{1t+1}}$; $c_{2t} = \dfrac{c'_{2t}}{\beta P_{1t+1}}$; i.e., c_{1t} and c_{2t} are costs in terms of the present value of P_{1t+1}.

The farmer's information set at time t is assumed to be

$$\Omega_t = \{A_{1t-1}, A_{1t-2}, \ldots A_{2t-1}, A_{2t-2}, \ldots \bar{A}_t, \bar{A}_{t-1},$$

$$\ldots P_{1t}, P_{1t-1} \ldots P_{2t}, P_{2t-1} \ldots W_t, W_{t-1}, \ldots \qquad (2.21)$$

$$c_{1t}, c_{1t-1}, c_{2t}, c_{2t-1}, \ldots a_{1t-1}, a_{2t-1}, \ldots\}$$

For the maximization of 2.20 to be a well-posed problem, it is necessary to be explicit about farmer's views about the laws of motion of the exogenous random variables that he cannot control, because these variables influence his choice of best land allocation. For problem (2.20), these exogenous or uncontrollable variables are $\bar{A}_t$, P_t, a_{1t}, c_{1t}, c_{2t} and W_t. Farmers care about the present and future behavior of the variables $\{P_t\ W_t\}$ because they influence the future behavior of output prices. Farmers care about the evolution of the variables $\{\bar{A}_t\ a_{1t}\}$ because they affect output directly through the production function. The process for the variables $\{c_{1t}\ c_{2t}\}$ affects the future course of costs of production. It is, therefore, necessary to assume that farmers know the processes by which these exogenous variables are generated.

The maximization of 2.20 is subject to a given level of A_{1t-1} and to laws of motion for the stochastic processes a_{1t}, P_t, $\bar{A}_t$, W_t, c_{1t} and c_{2t}. The shock to productivity (a_{1t}), the production costs (c_{1t} and c_{2t}) and the total cultivated land ($\bar{A}_t$) are assumed to be generated by the following stochastic processes:

$$\begin{aligned} \delta_a(L)\,a_{1t} &= U_t^a \\ \delta c_1(L)\,c_{1t} &= U_t^{c_1} \\ \delta c_2(L)\,c_{2t} &= U_t^{c_2} \end{aligned} \qquad (2.22)$$

$$\delta_A(L)\bar{A}_t = U_t^A;$$

where each disturbance U_t^a, $U_t^{c_1}$, $U_t^{c_2}$ and U_t^A is a white noise and L = lag operator, where $L^k X_t = X_{t-k}$.

$$\delta_A(L) = 1 - \sum_{j=1}^{r_A} \delta_{A_j} L^j, \text{ where } \delta_{A_j} \text{ is a scalar}$$

$$\delta_a(L) = 1 - \sum_{j=1}^{r_a} \delta_{a_j} L^j, \text{ where } \delta_{a_j} \text{ is a scalar}$$

$$\delta_{c_i}(L) = 1 - \sum_{j=1}^{r_{c_i}} \delta_{c_{ij}} L^j; \text{ where } \delta_{c_{ij}} \text{ (i=1,2) is a scalar.}$$

Let Z_t be a (q+1)x1 vector with P_t being the first element and the remaining q elements being W_t which helps predict P_t; i.e.,

$$Z_t = \begin{bmatrix} P_t \\ W_t \end{bmatrix}.$$

Assume that Z_t follows r_z^{th} order vector autoregressive process

$$\delta_z(L) Z_t = U_t^z \quad \text{where } U_t^z \text{ is a (q+1)x1} \tag{2.23}$$

vector of white noise and

$$\delta_Z(L) = I - \sum_{j=1}^{r_Z} \delta_{Z_j} L^j \quad \text{where}$$

δ_{Z_j} is a (q+1) x (q+1) matrix and I_Z is the identity matrix.

Each of the above stochastic processes, Equations (2.23) and (2.22) is assumed to be of exponential order less than

$1/\sqrt{\beta}$,[4] and all random variables have finite first- and second-order moments.

With these specifications, the maximization is now well-posed. The representative farmer maximizes (2.20) subject to the laws of motion for the stochastic processes (Equations 2.22 and 2.23) and the information available to him (Equation 2.21). Solutions to quadratic objective functions like (2.20) have special characteristics in the sense that they exhibit the "certainty equivalence" or separation principle (Sargent, 1979). The problem can be solved in two steps: first, solve the nonstochastic version of the optimizing problem; second, obtain the minimum mean squared error forecast of the exogenous variables, which are the conditional expectations, and replace the exogenous variables in the solution of the first step by their conditional expectations.

The first-order necessary conditions for maximization of (2.20) are the "Euler equations" and transversality condition. The following system of T stochastic "Euler equations" are derived by differentiating (2.20) with

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[4] Z_t is of exponential order less than $1/\sqrt{\beta}$ if for some $K>0$ and some x such that $1\leq x \leq 1/\sqrt{\beta}$, $|EZ_{t+j}|<K(x)^{j+t}$ for all t and $j\geq 0$ (see Sargent, 1979). This is a necessary condition for the transversality condition (defined below) to hold.

respect to A_{1t}, t = 0,1,2,..., T-1:

$$\beta^t[d_0+c_{2t}-c_{1t}+a_{1t}) - d_1A_{1t} + d_2\bar{A}_t - d_2A_{1t-1} - P_{t+1}]$$
$$- \beta^{t+1}d_2A_{1t+1} = 0 \qquad (2.24)$$

The transversality condition is:

$$\lim_{T\to\infty} \beta^T\{(d_0+c_{2T}-c_{1T}+a_{1T})-d_1A_{1t}+d_2\bar{A}_T-d_2A_{1T-1}-P_{T+1}\} = 0 \qquad (2.25)$$

Note that if $d_2=0$, Equation (2.24) and (2.25) are identical and Equation 2.20 is a static model with linear demand equations for land.

The system of second-order difference equations of A_{1t}, (2.24), can be written as

$$\beta d_2A_{1t+1}+d_1A_{1t}+d_2A_{1t-1} = (d_0+c_{2t}-c_{1t}+a_{1t}+d_2\bar{A}_t-P_{t+1})$$

or

$$\beta A_{1t+1}+\phi A_{1t}+A_{1t-1} = d_2^{-1}(d_0+c_{2t}-c_{1t}+a_{1t}+d_2\bar{A}_t-P_{t+1})$$

where

$$\phi = \frac{d_1}{d_2} \qquad (2.26)$$

To solve Equation (2.26), two boundary conditions are needed. One boundary condition is given by the initial value $A_{1,-1}$; and the other is given by the terminal conditions, Equation (2.25). Sufficient conditions for the terminal condition to hold are that each of the sequences $\{a_{1t}\}$, $\{c_{1t}\}$, $\{c_{2t}\}$, $\{\bar{A}_t\}$, $\{P_t\}$ and the solution for A_{1t} be of exponential order less than $1/\sqrt{\beta}$.

The necessary condition for an optimal solution for (2.20) is then satisfied if we can find a solution to the difference Equation (2.26) subject to the transversality conditions (2.25) and the initial value $A_{1,-1}$. To aid in obtaining a solution to Equation (2.26), it is rewritten using the lag operator L^j:

$$\beta(1+ \frac{\phi}{\beta}L + \frac{1}{\beta}L^2)A_{1t+1} = d_2^{-1}(d_0+c_{2t}-c_{1t}+a_{1t}+d_2\bar{A}_t-P_{t+1}). \tag{2.27}$$

To obtain a solution to Equation (2.27), we seek a factorization of the second-order polynomial in lag operator:

$$\begin{aligned}(1 + \frac{\phi}{\beta}L + \frac{1}{\beta}L^2) &= (1-\lambda_1 L)(1-\lambda_2 L) \\ &= 1-(\lambda_1+\lambda_2)L + \lambda_1\lambda_2 L^2 \end{aligned} \tag{2.28}$$

where λ_1 and λ_2 are the reciprocals of the roots of the polynomial $(1 + \frac{\phi}{\beta}L + \frac{1}{\beta}L^2) = 0$. Equating powers of L on both sides of 2.28, we have

$$-\frac{\phi}{\beta} = \lambda_1+\lambda_2 \tag{2.29}$$

$$\frac{1}{\beta} = \lambda_1\lambda_2, \text{ where } \phi = \frac{d_1}{d_2}$$

so solutions for λ_1 fulfill the condition:

$$\beta\lambda_1\lambda+ \lambda_1^{-1} = -d_1/d_2 \; . \tag{2.30}$$

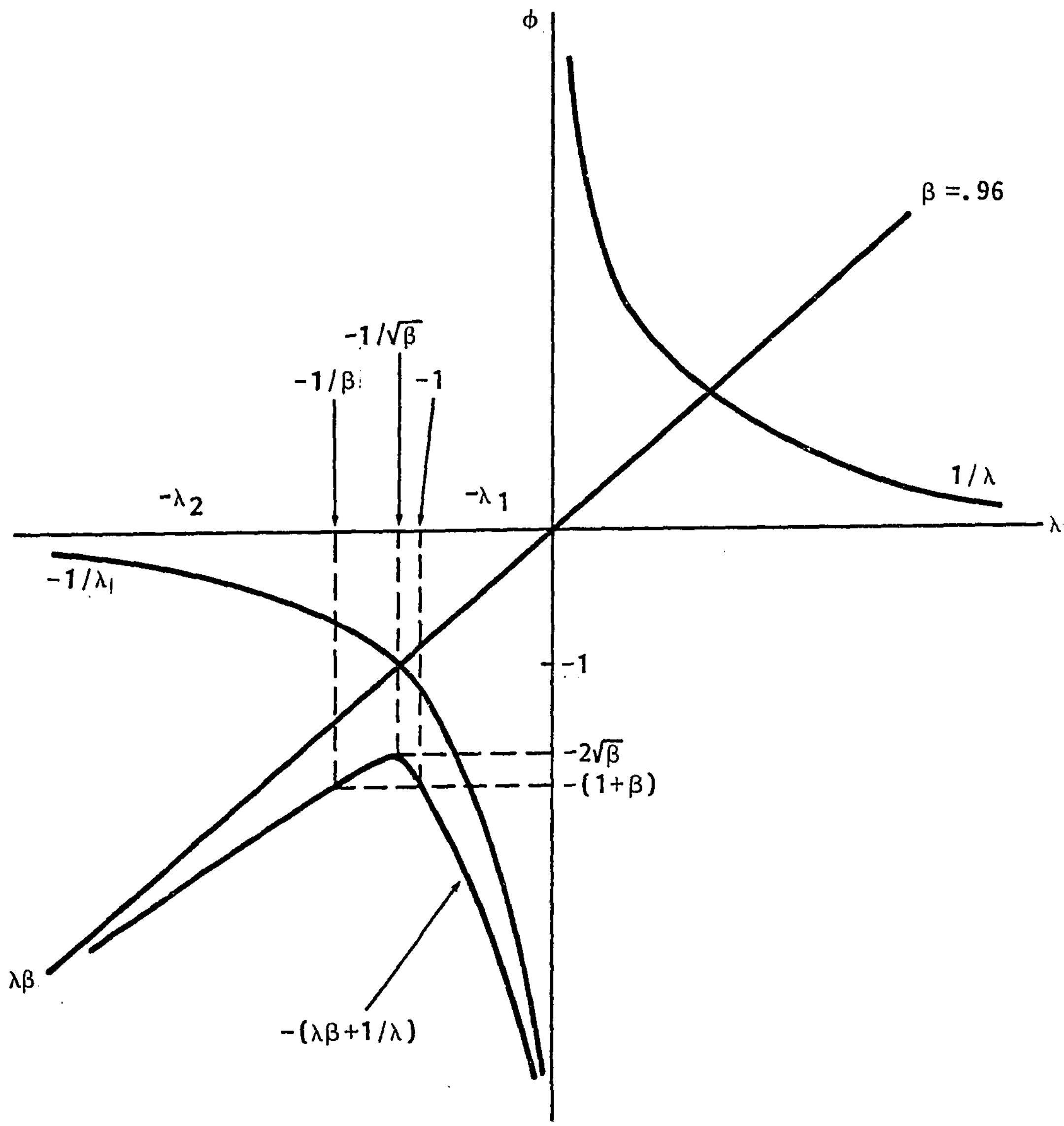

Figure 2.1. Graph of $\phi = -(\lambda\beta + \frac{1}{\lambda})$

Since $d_1/d_2 > 0$, the solution values for (2.30), occur in the 3rd quadrant of Figure 2.1 and are negative. Note that if λ_1 satisfies (2.30), so does λ_2; since $\lambda_2 = (\lambda_1\beta)^{-2}$. As Figure 2.1 shows, the function $-\phi = \beta\lambda_1 + \lambda_1^{-1}$ attains a maximum value of $-2\sqrt{\beta}$ at $\lambda_1 = -1/\sqrt{\beta}$. If $\lambda_1 = -\frac{1}{\sqrt{\beta}}$, then $\lambda_2 = (\lambda_1\beta)^{-1} = \frac{-1}{\sqrt{\beta}}$; hence $\lambda_1 = \lambda_2$. If $\frac{d_1}{d_2} > 2\sqrt{\beta}$, then the smaller root of $\lambda_1^{-1} + \lambda_1\beta$ must be greater than $-\frac{1}{\sqrt{\beta}}$; and λ_2 must be less than $-1\sqrt{\beta}$; i.e., $0 < |\lambda_1| < \frac{1}{\sqrt{\beta}} < |\lambda_2|$.

To obtain a stable solution for A_{1t}, we require that $|\lambda_1| < 1$ and this requires $\frac{d_1}{d_2} > 1+\beta$. Thus, the restrictions on λ_1 and λ_2 are

$$0 < |\lambda_1| < |1| < \frac{1}{\sqrt{\beta}} < |\lambda_2| = \frac{1}{\beta\lambda_1} .$$

We now rewrite Equation (2.27) as

$$\beta(1-\lambda_1 L)(1-\lambda_2 L)A_{1t+1} = d_2^{-1}[d_0+c_{2t}-c_{1t}+a_{1t}+d_2\bar{A}_t-P_{t+1}] \tag{2.31}$$

where $|\lambda_1| < 1$, $|\lambda_2^{-1}| < 1$

Applying the forward inverse of $(1-\lambda_2 L)$[5] to both sides of (2.10), we obtain

[5]The forward inverse of $(1-\lambda_2 L)$ is

$$(1-\lambda_2 L)^{-1} = \frac{1/\lambda_2 L}{\frac{1}{\lambda_2 L} - 1} = \frac{(\lambda_2 L)^{-1}}{1-(\lambda_2 L)^{-1}} .$$

$$(1-\lambda_1 L) A_{1t+1} = - \frac{(\beta d_2 \lambda_2 L)^{-1}}{1-\lambda_2^{-1} L^{-1}} \times$$

$$[d_0 + c_{2t} - c_{1t} + a_{1t} + d_2 \bar{A}_t - P_{t+1}]. \qquad (2.32)$$

Observe that $- \frac{(\lambda_2 L)^{-1}}{1-\lambda_2^{-1} L^{-1}} X_t = -\lambda_2^{-1} \sum_{i=0}^{\infty} (\frac{1}{\lambda_2})^i X_{t+1+i}$

and that $\lambda_2^{-1} = \lambda_1 \beta$, thus, the solution of Euler's equation is

$$A_{1t+1} = \lambda_1 A_{1t} - \frac{\lambda_1}{d_2} \cdot \sum_{i=0}^{\infty} (\lambda_1 \beta)^i [d_0 + c_{2t+1+i} - c_{1t+1+i}$$

$$+ a_{1t+1+i} + d_2 \bar{A}_{t+1+i} - P_{t+2+i}] \qquad (2.33)$$

or

$$A_{1t} = \lambda_1 A_{1t-1} - \frac{\lambda_1}{d_2} \sum_{i=0}^{\infty} (\lambda_1 \beta)^i [d_0 + c_{2t+i} - c_{1t+i} + a_{1t+i}$$

$$+ d_2 \bar{A}_{t+i} - P_{t+1+i}] \qquad (2.34)$$

Applying the certainty equivalence principle to Equation (2.34), the optimal solution to Equation (2.20) is

$$A_{1t} = \lambda_1 A_{1t-1} - \frac{\lambda_1}{d_2} \sum_{i=0}^{\infty} (\lambda_1 \beta)^i [d_0 + E(c_{2t+i}) - E(c_{1t+i})$$

$$+ E(a_{1t+i}) + d_2 E(\bar{A}_{t+i}) - E(P_{t+1+i})] \qquad (2.35)$$

Thus, the optimal land allocation at time t depends, among other things, on the land allocation at time t-1 and all future values of P_t weighted by a factor which depends on the production parameters of the model and the discount factor.[6]

Equation (2.35), however, cannot be a decision rule because the expected values of the random variables c_{1t+i}, c_{2t+i}, a_{1t+i}, $\bar{A}_{t+i}$, P_{t+1+i} are not known to farmers at time t. If farmers form rational expectations about these variables, they make predictions of these variables that are conditional on their information. These predictions are the same as the conditional mathematical expectations of the variables which depend on the stochastic processes generating them.

We use the Weiner-Kolmogorov formula to obtain the conditional expectations of the exogenous variables; i.e., the Wiener-Kolmogorov prediction formula is applied to express $E_t\, X_{t+j}$ as a function of lagged values of X_t.[7] If the

[6] Recall that λ_1 is a function of the production function parameters and the discount factor.

[7] The Weiner-Kolmogorov formula is:

$$E_t X_{t+j} = \left[\frac{\delta_x(L)}{L^j}\right]_+^{-1} \cdot \delta_x(L) X_t \qquad (2.36)$$

where

$$\left[\sum_{j=-\infty}^{\infty} \alpha_j L^j\right]_+ = \sum_{j=0}^{\infty} \alpha_j L^j .$$

Using (2.36), we express $E(\cdot)$ in (2.35) in terms of known variables (Appendix A).

exogenous variables in right-hand side of (2.35) have a finite order autoregressive representation, then, from the results of Appendix A, the land allocation decision rule can be written as a function of lagged land allocation and current and lagged values of the exogenous variables:

$$A_{1t} = \Pi_0 + \lambda_1 A_{1t-1} + \Pi_1(L)\bar{A}_t + \Pi_2(L)P_t + \Pi_3(L)c_{1t} + \Pi_4(L)c_{2t} + \Pi_5(L)a_{1t} + \Pi_6(L)w_{1t} + \ldots + \Pi_{q+5}(L)w_{q_t} \quad (2.37)$$

where $\Pi_i(L)$ is a finite order polynomial in the lag operator which depends on the order of the autoregressive process of the variable. The Π's are nonlinear functions of the production function parameters, the discount factor and parameters of the laws of motion for the stochastic process $\{\bar{A}_t, P_t, c_{1t}, c_{2t}, a_{1t}, w_t\}_0^\infty$. These nonlinear functions imply cross-equation restrictions on the parameters of decision rule. Because the parameters in (2.37) are functions of the parameters in the farmers objective function and the parameters of the stochastic processes of the exogenous variables which includes government policy variables, Equation (2.37) is not invariant to governmental policy. Because of potential land fertility deterioration, Equation (2.37) also exhibits first-order negative serial co-relation ($\lambda_1 < 0$) in land allocation.

Unlike the traditional supply response models, a change, for example, in the price process affects the structure of the correlation between the right-hand-side variables in (2.37) and land allocation. To see this point and the effect of a price changes on land allocation, consider the following specification for uncertain exogenous variable:

$$P_t = \alpha_1 P_{t-1} + \alpha_2 G_{t-1} + U_t^p, \quad |\alpha_1|<1 \tag{2.38a}$$

$$\bar{A}_t = \gamma_1 \bar{A}_{t-1} + \gamma_2 \bar{A}_{t-2} + U_t^A \tag{2.38b}$$

$$G_t = gG_{t-1} + U_t^G, \quad |g| < 1 \tag{2.38c}$$

$$a_{1t} = a_{1t-1} + U_t^a, \quad |\rho| < 1 \tag{2.38d}$$

$$c_{1t} = c_{2t} = 0$$

where U_t^P, U_t^A, U_t^G and U_t^a are iid with zero mean and constant variance; and the roots of $|1+\gamma_1 x+\gamma_2 x^2| = 0$ lie outside the unit circle. W_t contains only one variable G_t, government policy variable (e.g., government price support).

Using the results of Appendix A and Equations 2.38a-2.38d), Equation (2.35) reduces to the following land allocation decision rule

$$A_{1t} = \Pi_0 + \lambda_1 A_{1t-1} + \Pi_1 \bar{A}_t + \Pi_2 \bar{A}_{t-1} + \Pi_3 P_t$$
$$+ \Pi_4 G_t + \Pi_5 a_{1t-1} \tag{2.39}$$

where

$$\Pi_0 = -\frac{\lambda_1}{d_2} \cdot \frac{d_0}{(1-\lambda_1\ )}$$

$$\Pi_1 = -\frac{\lambda_1}{(1-\gamma_1\lambda_1\beta-\gamma_2(\lambda_1\beta)^2)} \qquad (2.40)$$

$$\Pi_2 = -\frac{\lambda_1^2\beta\lambda_2}{(1-\gamma_1\lambda_1\beta-\gamma_2(\lambda_1\beta)^2)}$$

$$\Pi_3 = \frac{\lambda_1}{d_2} \cdot \frac{\alpha_1}{(1-\lambda_1\beta\alpha_1)}$$

$$\Pi_4 = \frac{\lambda_1}{d_2} \cdot \frac{\alpha_2}{(1-g\lambda_1\beta)(1-\alpha_1\lambda_1\beta)}$$

$$\Pi_5 = -\frac{\lambda_1}{d_2} \cdot \frac{\rho}{(1-\rho\lambda_1\beta)}$$

$$\lambda_1^{-1} = -\frac{d_1}{d_2} - \lambda_1\beta$$

$$|\alpha_1| < 1,\ |\rho| < 1,\ |g| < 1,\ |\gamma_1| < 1,\ \gamma_1+\gamma_2 < 1,\ \gamma_2-\gamma_1 < 1.$$

Equation (2.40) shows the restrictions across Equations (2.38) and (2.39) as well as the restriction within Equation (2.39). These restrictions are restrictions implied by rational expectations hypothesis for this particular case. In general, Equation (2.37) characterizes the land allocation decision rule.

Supply elasticities

Both long run and short run supply elasticities are defined for the model. Let X_t be one of the variables beyond the control of the farmer. Following Eckstein (1983), we define two types of elasticities:

(1) Elasticity of expected output with respect to an expected change in X_t; and

(2) Elasticity of actual output with respect to an unexpected change in X_t. The long run elasticity of expected output (acreage) with respect to an expected change in X_t, $\bar{\eta}_x$, measured at the sample mean is defined as

$$\bar{\eta}_x = \frac{\partial E(A_1)}{\partial E(x)} \cdot \frac{\bar{X}}{\bar{A}_1}$$

The short run elasticity of expected output with respect to X_t is defined as

$$\eta_x = \frac{\partial E_t(A_{1t})}{\partial E_t(X_{t+j})} \cdot \frac{\bar{X}}{\bar{A}_1}$$

The long-run elasticity measures the effect of an expected change in the mean of X_t on the mean of output (land allocation); and the short-run elasticity measures the effect of an expected change in X_t, j periods ahead, on the current land allocation. From Equation (2.35), the short-run elasticity of supply with respect to price is

$$\eta_p = \frac{\lambda_1}{d_2} \cdot (\lambda_1 \beta)^i \cdot \frac{\bar{P}}{\bar{A}_1}$$

To calculate the long-run supply elasticity with respect to price, ignore, without loss of generality, the other terms in the right-hand side of (2.35) so that (2.35) becomes

$$A_{1t} = \lambda_1 A_{1t-1} + \frac{\lambda_1}{d_2} \sum_{i=0}^{\infty} (\lambda_1 \beta)^i E(P_{t+1+i}) \qquad (2.35a)$$

A_{1t} is a stationary time series, and taking expectations of both sides of (2.35a):

$$(1-\lambda_1)E(A_1) = \frac{\lambda_1}{d_2} \cdot \frac{1}{(1-\lambda_1\beta)} \cdot E(P)$$

and

$$\bar{\eta}_p = \frac{\lambda_1}{d_2(1-\lambda_1)(1-\lambda_1\beta)} \cdot \frac{\bar{P}}{\bar{A}_1}$$

The long-run supply elasticity for crop 1 is negative but the short-run supply elasticity may be either positive or negative.[8] The long-run supply elasticity of crop 1 is negative because $\lambda_1<0$, $d_2>0$, $1-\lambda_1>0$ and $1-\lambda_2\beta>0$, given that the price of crop 1 is in the denominator of P_t or $\bar{P}$. However, the short-run elasticity oscillates in sign because of the alternating sign of $(\lambda_1\beta)^i$. The magnitude of the

[8]These elasticities (long-run and short-run) depend on the production function parameters and the discount factor.

elasticities declines for a change in expected price that is further away in time. This alternating signs in the short-run supply elasticities can be explained by the crop technology (crop rotation) incorporated in the model. If at t the price of crop 2 relative to the price of crop 1 is expected to be higher at harvest time (t+1), farmers will plant more of crop 2 or less of crop 1 in t. This is consistent with the partial derivative of A_{1t} with respect to P_{t+1} being negative; i.e., $\frac{\partial A_{1t}}{\partial E(P_{t+1})} = \frac{\lambda_1}{d_2} < 0$.

If, at t, the price of crop 2 relative to the price of crop 1 is expected to be higher 2 time periods ahead or at t+2, farmers plant more acres of crop 1 at time t so that the yield of crop 2 when planted at time t+1 will be larger. Thus,

$$\frac{\partial A_{1t}}{\partial E(P_{t+j})} = \frac{\lambda_1}{d_2} (\lambda_1 \beta)^{j-1} \lessgtr 0 \quad \text{as } j \text{ is } \begin{cases} \text{odd} \\ \text{even} \end{cases} \qquad j = 1,2,\ldots$$

Unlike the elasticities with respect to expected changes, the elasticities with respect to an unexpected change in prices depend not only on the production parameters and the discount factor, but also on the parameters of the price process. In other words, the computation of elasticities of supply with respect to an unexpected change in prices requires complete identification of the parameters of the decision rule for land allocation. To see this, we first define elasticities with respect to an unexpected change in a variable. Let $U_t^x = X_t - E(X_t)$ be an unexpected change (shock) to X_t at time t. Suppose

$$U_{t+k}^x = \sigma_x = \sqrt{var(U_t^x)}, \quad K = 0$$
$$= 0 \qquad K \neq 0$$

The elasticity response of output, K periods ahead, with respect to an unexpected once-but-not-for-all one standard deviation shock in X_t is defined as (Eckstein, 1983)

$$\gamma(k) = \frac{\hat{A}_{1t+k} - E(A_{1t})}{\sigma_x - E(X_t)} \cdot \frac{\bar{X}}{\bar{A}_1} ;$$

where $\hat{A}_{1t+k}$ is the value of land allocation at time t+k and $A_{1s} = E(A_{1t})$ for s<t. We note that since $|\lambda_1|<1$, the A_{1t} process is stationary so that in the long run $\hat{A}_{1t+s} \rightarrow E(A_{1t})$ and $\gamma(s) \rightarrow 0$. In order to calculate the elasticity with respect to an unexpected shock in prices, consider the decision rule, with other variables ignored, and the price process.

$$A_{1t} = \lambda_1 A_{1t-1} + \Pi_3 P_t + U_t^{A_1}, \quad |\lambda_1|<1$$

$$P_t = \alpha_1 P_{t-1} + U_t^p, \quad |\alpha_1|<1.$$

Let

$$U_t^A = 0, \quad \forall t$$

$$\bar{A}_{1t} = A_{t-1} = P_{t-1} = \bar{P}_t = 0.$$

Now suppose there is a shock in P_t at time t with the following property:

$$U_t^p = \sigma_p, \ U_s^p = 0, \quad \forall s \neq t$$

so

$$P_t = \sigma_p$$

$$P_{t+k} = \alpha_1^k \sigma_p$$

and

$$A_{1t} = \Pi_3 \sigma_p$$

$$A_{1t+1} = \Pi_3 \sigma_p (\lambda_1 + \alpha_1)$$

$$A_{1t+2} = \Pi_3 \sigma_p [\lambda_1(\lambda_1+\alpha_1)+\alpha_1^2]; \quad \text{etc.}$$

Thus, response of crop acreage to a one standard deviation unit shock in prices depends not only on the parameter of the production function and the discount factor (through λ_1 and Π_3) but also on the parameter of the price process (α_1). Using the above results, the elasticity response of output to an unexpected shock in prices, $\gamma(s)$'s, can be calculated. Further, we observe that since $|\lambda_1|<1$ and $|\alpha_1|<1$, $A_{1t+k} \to 0$ as $K \to \infty$. The above sequence of A_{1t} follows a cobweb cycle. If Π_3 is negative, A_{1t} is negative; A_{1t+1} is positive and so on.

From the preceding discussion, it is clear that to see the response of output to some changes in prices we need a complete identification of the models parameters. However, if the aim is to estimate supply elasticities with respect to expected change in prices, we need to know the production parameters only (the discount factor being given).[9]

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[9]See Eckstein (1983) for details about this point.

CHAPTER III. PRELIMINARY DATA ANALYSIS

In this chapter, an empirical model is proposed and preliminary tests are performed. The model proposed is a vector autoregression. Some of the assumptions made in Chapter II are tested with this model, e.g., exogeneity of prices. A moving-average representation is also derived from the vector autoregression in order to analyze the responses of land allocations to shocks in prices and vise versa.

In the first section of this chapter, the unrestricted vector autoregressive model (VAR) is described and formulated. The VAR is then used to derive, by simulation, a moving average representation (MAR) which provides a convenient framework from which a general description of dominant characteristics of the variables in the vector autoregression can be developed. This aspect of the analysis is presented in the second section. The results of the VAR and MAR analysis when applied to aggregate time series data on acreages, yields and relative price for the state of Iowa are presented in the third section. The last section presents some tests of stability of the model over the sample period.

The Unrestricted Vector Autoregressive Model

In this section, an unrestricted reduced form of an econometric model following Sims (1980), Sargent (1978b), Eckstein (1981) and Falk (1982) is formulated. These models are called unrestricted in the sense that no restrictions based on a priori knowledge is imposed on coefficients and all variables are treated as endogenous. When these models are fitted to multivariate time series data, they may suggest feedback relationships that might be incorporated in the macroeconometric modelling. In what follows, a VAR model is formulated for the time series of land allocation outputs and prices.

Land allocation and output are correlated over time. Output decisions are based on anticipated or expected future prices which are mainly a function of past prices. Thus, prices and output should be correlated. Anticipation of a higher price for corn relative to soybeans should lead to a larger area being planted to corn and larger total corn production. Unexpected increases in output may have a downward pressure on prices. To analyze the nature of these feedbacks between prices and output or land allocation it is convenient to present the time series of output/land allocation and prices as a vector process.

Let X_t be a nx1 vector of output or land allocation and let P_t be an mx1 vector of relevant relative prices. Define $Y_t = (X_t, P_t)$ and assume the (n+m)-dimensional vector Y_t is covariance-stationary.[1] The vector Y_t can be regarded as an n+m dimensional, covariance-stationary stochastic process. Furthermore, let Y_t be arbitrarily well-approximated (in the mean-square sense) by the following r^{th} order vector autoregressive process:

$$Y_t = A_1 Y_{t-1} + A_2 Y_{t-2} + \ldots + A_r Y_{t-r} + U_t. \tag{3.1}$$

Equation (3.1) can be expressed as

$$Y_t = \sum_{s=1}^{r} (A_s) Y_{t-s} + U_t \tag{3.2}$$

or

$$Y_t = A(L) Y_t + U_t \tag{3.3}$$

where

$$A(L) = A_1 L + A_2 L^2 + \ldots + A_r L^r.$$

Y_t is an (n+m)x1 vector of random variables, A(j), j = 1...r, are (n+m)x(n+m) matrices of time-invariant coefficients, i.e., A_s in (3.2) depends on s but not on t. This follows from the stationarity of Y_t.

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[1] X_t is cov-stationary if: (1) The expected value of X_t is constant for all t, (2) the covariance matrix of $(X_{t_1}, X_{t_2}, \ldots, X_{t_n})$ is the same as the covariance matrix of $(X_{t_1+h}, X_{t_2+h}, \ldots X_{t_n+h})$ for all nonempty finite sets of indices $(t_1, t_2, \ldots, t_n)$ and all h such that $t_1, t_2, \ldots, t_n$, $t_1+h, t_2+h, \ldots, t_n+h$ are contained in the index set (Fuller, p. 4).

U_t is an (n+mxl) "innovation vector" in the Y_t process. The innovation of a stochastic process is that part of the process which cannot be predicted on the basis of information available from the past. This definition implies that the expected value of the current innovation, conditional on the past information, is zero, and the innovation process is serially uncorrelated. Thus, $\hat{Y}_t = A_1 Y_{t-1} + A_2 Y_{t-2} + \ldots + A_r Y_{t-r}$ is the "best" predictor of Y_t.

The model (Equations 3.1-3.3), is unconstrained in the sense that, a priori, each of the components of the vector process Y_t is assumed to be endogenous with respect to the other components of the process and the lag structure is symmetric across the variables and equations of (3.2). In other words, none of the components of the A_s matrices, where

$$A_s = \begin{bmatrix} \underset{n \times n}{A_{11}(s)} & \underset{n \times m}{A_{12}(s)} \\ \underset{m \times n}{A_{21}(s)} & \underset{m \times m}{A_{22}(s)} \end{bmatrix},$$

are assumed to be zero a priori. Given this specification for the system of equations, Zellner's seemingly unrelated regression methods is appropriate for estimating the A's. Because the same explanatory variables (RHS variables) are used in each equation, ordinary least-squares estimation applied equation by equation to each equation in the Y_t vector process is equivalent to applying generalized least squares (Theil, 1971; Kmenta, 1971; and Judge et al., 1980).

An appropriate lag length for 3.1 must be determined. This can be accomplished by performing a statistical test that a subset of the A_j's are zero. Consider the following two specifications of (3.1).

$$Y_t = A_1 Y_{t-1} + \ldots + A_{r_1} Y_{t-r_1} + U_{1t} \quad (3.1a)$$

$$Y_t = A_1 Y_{t-1} + \ldots + A_{r_2} Y_{t-r_2} + U_{2t} \text{ where } r_1 < r_2 \quad (3.1b)$$

The system (3.1a) can be viewed as a restricted version of (3.1b), the restriction being $A_s = 0$, $s = r_1+1, \ldots r_2$. Under this null hypothesis, the likelihood ratio statistic is $T\,(\text{Log}|Dr| - \text{Log}|D_u|)$ which has a $\chi^2(q)$ as its asymptotic distribution. D_u and D_r are the sample covariance matrices for the unrestricted and the restricted system, respectively; T is the sample size; and q is the total number of restrictions tested. To account for some bias which is believed to be inherent in such tests (Sims, 1980, p. 17), Sims modified this test by using (T-K) rather than T for calculating the test statistic; where K is equal to the number of coefficients per equation in the unrestricted system.

The modified test statistic is employed in this study. Tests will be confined to cases for $r_2 - r_1 = 1$, and an increase by one of the lag length increases the number of unknown parameters by $(n+m)^2$.

The VAR of the Y_t process, as Sims (1980) and Falk (1982) have argued, is hard to interpret. The reason is that VARs will generally be characterized by oscillating signs of coefficients on successive lags of a variable and complicated cross-equation feedbacks which are difficult to untangle. However, VAR of Y_t can be employed to perform informative statistical tests about the nature of economic relationships among the variables in Y_t. These tests are:

(i) X_t does not Granger-cause P_t[2]. In Chapter II, output prices were assumed to be exogenous. For this to be the case, X_t must not Granger-cause P_t. If X_t does not Granger-cause P_t, $A_{21}(s)$ must equal zero for all $s = 1,2,\ldots r$. Therefore, testing the hypothesis that price is not Granger-caused by output/land allocation is equivalent to testing for $A_{21}(s) = 0$. Since this test is performed on only one equation, the price equation, an F-statistic is employed.

(ii) No structural change during the sample period.

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[2]Granger-causality is defined below.

This test can be executed by adding dummy variables to permit coefficients to be different for each variable in different sample periods. The test statistic for each component of Y_t is an F-statistic while for the vector process Y_t, the test-statistic is the modified likelihood ratio statistic.

The Moving Average Representation

The autoregressive system like Equation (3.1) are difficult to describe succinctly. It is especially difficult to understand and interpret the estimated coefficients. The estimated coefficients on successive lags tend to oscillate, and there are complicated cross-equation feedbacks. A common practice is to derive the MAR of the VAR and examine how the system of variables respond to shocks. As a result, the MAR will generally be a more convenient device to provide an economic interpretation to the estimated system.[3] The derivation MAR from a VAR and the description of the MAR are the objectives of this section.

Examination of how each component of the VAR system responds over time to shocks originating from various sources within the system, will give an insight to the dominant feedbacks among the components over the sample periods. For example, if prices are exogenous, with respect to output, the

[3]For a detailed discussion of this point, see Sims (1980).

time path of prices should not be very responsive to shocks originating from output. The shocks, which are considered, are residuals from the equations of the VAR, i.e., the innovations in the systems dependent variable. In particular, it is interesting to see the responses of land allocation to shocks in crop prices and the response of prices to shocks in land allocations. Before proceeding further with the discussion of the MAR, it seems useful to describe the derivation of the MAR from VAR.

Recall from (3.3):

$$Y_t = \sum_{s=1}^{r} A_s Y_{t-s} + U_t \tag{3.3a}$$

where the roots of the characteristic equation $\det\{I-A(Z)\} = 0$ exceed one in absolute value (a necessary condition for Y_t to be stationary), and U_t is the vector of innovations in the Y_t process. These conditions guarantee that Y_t has a moving average representation, i.e.,

$$Y_t = \sum_{s=0}^{\infty} B_s U_{t-s} = \sum_{s=0}^{\infty} B_s L^s U_t \tag{3.4}$$

From (3.3), $[(I-A(L)]Y_t = U_t$

or

$$Y_t = (I-A(L))^{-1} U_t$$

Therefore, $\Sigma B_s L^s = (I-A(L))^{-1}$; which exists since Y_t is assumed to be stationary.

Finding the B coefficients is equivalent to inverting the matrix polynomial [I-A(L)]. It will be shown that except for scaling, tracing out the response of (3.1) to typical random shocks is equivalent to deriving the moving average representation by matrix polynomial long division.

The coefficients of the VAR, are obtained by estimating (3.1) after the appropriate lag length has been determined. Denote the individual elements of Y_t' as follows:

$$Y_t' = (X_{1t}, X_{2t}, \ldots, X_{nt}, P_{1t}, P_{2t}, \ldots, P_{mt})'$$
$$= (Y_{1,t}, Y_{2,t}, \ldots, Y_{n,t}, Y_{n+1,t} \ldots, y_{n+m,t})' \qquad (3.5)$$

and after estimating VAR(3.1), consider a set of initial conditions in which $y_{j,0} = 1$ and all other elements of Y_0 are set equal to zero and $Y_k = 0$, $k \neq 0$. This initial condition is equivalent to setting $u_{j,0} = 1$ and all other elements of $U_0 = (u_{1,0}, u_{2,0}, \ldots, u_{j,0}, \ldots, u_{n+m,0})$ equal to zero, and $U_k = 0, k \neq 0$. Simulating the VAR, (3.1), to these initial conditions, will give the j^{th} column of the matrix B_s of Equation (3.4) which shows the responses, s period ahead, of each variable of the model, to an initial shock to the j^{th} variable. Through successive simulations in which the initial values of all the residuals in (3.1), except for one, are set equal to zero, all the components of B_s of the MAR (3.4), can be obtained. The i,j^{th} element of

B_s, $b_{ij}(s)$, which is obtained by such simulation is equivalent, except for scaling to $y_{i,0}$, $y_{i,k}$, $y_{i,2}, \ldots y_{i,T}$.

Therefore, one can regard the i,j^{th} component of B_s, $b_{ij}(s)$, as the response s periods ahead of the i^{th} variable to an initial shock in the j^{th} variable. However, problems arise if the elements of the innovation vector U_t are contemporaneously correlated. The above simulation does not take this correlation into consideration. It will be useful to analyze the degree to which each innovation contributes to the overall variance in each variable. However, the presence of substantial contemporaneous correlation among the innovation vector makes it difficult to uniquely decompose the variance of the y_i's in this manner.

This problem can be circumvented. Consider a matrix H such that $V_t = HU_t$ has a variance-covariance matrix equal to the identity matrix. To obtain H, apply a Choleski decomposition to the variance-covariance matrix of U_t. Let M denote the contemporaneous covariance matrix of the innovation process, U_t. Since M is symmetric positive definite, M can be decomposed into M = GG' where G is a lower triangular (n+mxn+m) matrix and G' is its transpose.[4] Define a

[4]Using triangular decomposition means that, in general, the matrices G and G' will vary with the ordering of $y_1, y_2, \ldots, y_{m+n}$. Suppose m+n=4 and the variables are ordered as y_4, y_2, y_1, y_3. Then the transformation would imply that innovations in y_4 are instantaneously

vector process $V_t = G^{-1}U_t$, then V_t is serially uncorrelated and its elements are contemporaneously uncorrelated; i.e., $E[V_t V_t'] = E[G^{-1}U_t U_t' G^{-1'}] = E[G^{-1}GG'G^{-1'}] = I$. Further, since $U_t = GV_t$, the MAR, (3.4), can be written as

$$Y_t = \sum_{s=0}^{\infty} B_s G V_{t-s} \ . \qquad (3.6)$$

The MAR, (3.6), that has coefficients equivalent to (3.4) can be derived as follows: Having obtained G from the estimated variance-covariance matrix of the VAR's residual process, $B_s G$, s = 0,1,2,... can be obtained from the above simulation procedure. For some j, j = 1,2,...,m+n, set $y_{j,0}$ equal to one and all the other elements of Y_0 equal to zero. Then, premultiply this specification of Y_0 by G to obtain a new initial value of Y_0. This new initial value of Y_0 takes into account the possibility that the shock originating in variable j is being instantaneously reflected in other variables of the model. Holding $U_1, U_2, \ldots$, equal to their unconditional mean value of zero, the response of $Y_1, Y_2, \ldots$ to these initial conditions are obtained by

(footnote continued from p. 63)

reflected in all the three variables in the system. The innovation in y_2 instantly affect all the variables in the system except for y_4, and so on. Finally, innovation in y_3 affect y_3, but they have no immediate impact on any of the other variables (although, of course, through the pressure of lagged y_3's, they will eventually affect the rest of the system). Since this ordering is essentially arbitrary, several should be tried.

simulation.

The interpretation given to the component of the MAR, B_s, can be applied to the component of B_sG. In particular, the sum of the squares from s=0 to s=k of the i,j^{th} element of B_sG represents the part of the error variance in the k+1 step ahead forecast of y_i which is accounted for by innovation in y_j at s=0.

Define $h_i(t)$ as:

$$h_i(t) = \frac{\sum_{s=0}^{k} \tilde{b}_{ij}^2(s)}{\sum_{s=0}^{k} \sum_{j=1}^{m+n} \tilde{b}_{ij}^2(s)} \qquad (3.7)$$

where $\tilde{b}_{ij}(s)$ is the i,j^{th} component of B_sG. Equation (3.7) represent the proportion of the k+1 step-ahead forecast error variance in y_i attributable to shocks in y_j.

In particular, it is interesting to analyze output/land allocation responses to shocks in prices and vise versa. Exogeneity of prices can be evaluated by computing the percentage of forecast error in prices accounted for by innovations in output or land allocation. For price exogeneity assumption to be plausible, innovations in output or land allocation should account for only a small share of the forecast errors in prices.

Results

In the first section of this chapter, a VAR for land allocation/output and prices was formulated and in the second section a method of deriving the MAR was outlined. The procedures outlined in these sections are applied to the state data on acreages, yields and prices of corn and soybeans from Iowa.[5] In this section, the main results of the vector autoregression for the relative price (P_t), yield of soybeans (YS_t), yield of corn (YC_t), area (acreage) of soybeans (AS_t) and area (acreage) of corn (AC_t) are presented. Each of the variables AS_t, p_t, YC_t and YS_t is filtered with a constant and a linear trend. The variable AC_t is filtered with a constant, a linear trend and a dummy variable to account for governmental policies on feed grains (see Chapter IV). These filtered variables are presented by a vector Y_t; $Y_t' = (P_t, YS_t, YC_t, AC_t, AS_t)$. A test of the null hypothesis that the lag length was four for the VAR against the alternative hypothesis of five could not be rejected at the 5 percent significant level (Appendix C).

A test for exogeneity of prices is performed with the aid of Granger's causality tests (Granger 1969).

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[5]The data are aggregates for the state of Iowa for the years 1948-1980. The data set is given in Appendix E.

Definition:

Granger causality in the bivariate case: Consider two univariate stochastic processes Y_t and Z_t. The process Y_t is said to Granger-cause the process Z_t if, in a one-sided (population) projectionn of Z_t on past Z_t's and past Y_t's, past Y_t's "matter". Or Y_t

Granger causes Z_t if: (1) $Z_t = a(L)Z_t + b(L)y_t + v_t$; where

(i) $a(L) = a_1L + a_2L^2 + \ldots$

(ii) $b(L) = b_1L + b_3L^2 + \ldots$

(iii) V_t is innovative in Z_t process; and

(iv) b_j is nonzero for some $j \geq 1$.

A similar definition holds for the multivariate case. Granger-causality and econometric exogeneity are related in the following way in the bivariate case: For two univariate stochastic processes Y_t and Z_t, if Y_t Granger-causes Z_t, then Z_t is not econometrically exogenous with respect to Y_t in an equation expressing Y_t as a one-sided distribution lag of Z_t. If Y_t fails to Granger cause Z_t, then there will exist a representation of Y_t expressed as a one-sided distributed lag of Z_t in which Z_t is strictly exogenous. The failure of Y_t to Granger-cause Z_t is a necessary and sufficient condition for existence of a relationship in which Y_t is expressed as a distributed lag of Z_t and in which Z_t is strictly exogenous.

The following causality hypotheses concerning acreages, yields and prices of corn and soybeans were tested:

(i) Area of corn, area of soybeans, yield of soybeans and yield of corn each has zero coefficients in a fourth-order autoregressive equation for the relative price P_t. When AC, AS, YS and YC are each excluded from the price equation, the F-values are 2.44, 1.34, 1.58 and 1.86, respectively. The critical value of $F(4,9)$ at 5 percent significance level is 3.09. The exclusion of these variables from the price equation is supported.

(ii) Areas (corn and soybeans jointly) and yields (corn and soybeans jointly) have zero coefficients in the equation for the relative prices. When areas (jointly) are excluded from the price equation, the sample F-value is 1.72; and when yields (jointly) are excluded, the F-value is 1.23. The tabular value for $F(8,9)$ is 3.23. Thus, the hypothesis of zero coefficients for yields and zero coefficients for areas in the price equation cannot be rejected.

(iii) All the variables except lagged prices, in the right-hand-side of the price equation of (3.10) have zero coefficients. A price equation containing four lags of price (lagged areas and lagged yields excluded) is tested against a price equation containing four lags of the five variables. The result has an F-value of 2.28 with a critical value of

$F(16,9) = 3.00$. Thus, lagged areas and lagged yields can be excluded from price equation containing lagged prices, lagged areas and lagged yields of corn and soybeans.

(iv) Price has a zero coefficient in the equation for areas of corn and soybeans in Equation (3.1). When price is excluded from the corn area equation, the sample F-value is 4.2. When price is excluded from the equation for area of soybean, the sample F-value is 5.1. The tabular value of $F(4,9)$ at 5 percent significance is 3.63. Thus, the hypotheses of price exclusion from the separate equations for area of corn and area of soybeans are rejected.

(v) Acreage (AC and AS) equations have jointly zero coefficients for lagged price. When the four lags of price are excluded from area equations, the sample value of $X^2(8)$ is 15.91 with the critical value of $X^2(8)$ at 5 percent being 15.51. We reject the hypothesis of excluding price from the acreage equations.

The statistical tests (i-v) imply that while prices Granger-cause areas (acreages), areas do not Granger-cause prices. Thus, the tests support the assumption of exogeneity of prices.

From the estimated five dimensional vector of autoregressive equation for AS, AC, P, YS and YC, the moving-average representation (MAR) is obtained by simulating the estimated equation using the procedure outlined in 3.3-3.7. The MAR coefficients are equivalent to the responses of the VAR (3.1) to a random shock. The random shocks employed in the simulation are the residuals of the VAR. The responses of each variable in the five-dimensional vector process the shocks within the system appear in Appendix D.

Figures 3.1, 3.2 and 3.3 show the responses of AS(RAS) and responses of AC(RAC) to innovations (one standard deviation shock) in prices, yield of corn and area of corn, respectively (figures for responses to other innovations are given in Appendix D). In all cases, the order of variables is (P YC YS AC AS).[6] A price shock increases the area of soybean and decreases the area of corn in the first period. An interesting phenomenon is the cyclical movement of land allocations in response to innovations. This feature exists for almost all innovations. The other interesting feature is that the area of corn and soybean respond in opposite directions in most of the cases. The responses of both area of soybeans and area of corn to innovations in yields persist over a longer period.

The responses of price to innovation in corn and soybeans areas are given in Figures D.3 and D.4 (Appendix

[6]Changing the order to (P YS YC AS AC) did not change the result very much.

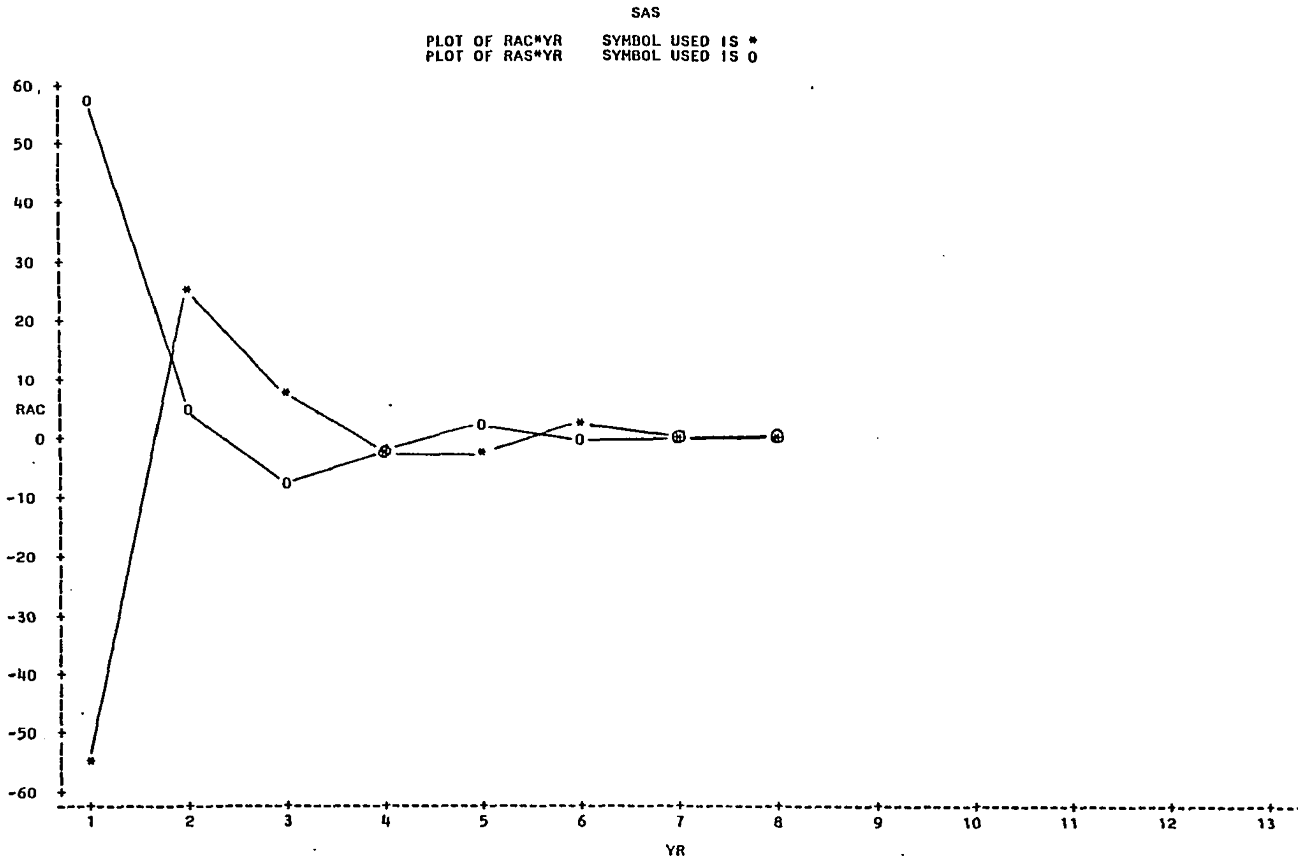

Figure 3.1. Plot of responses of area of corn and area of soybean to one standard deviation shock in prices

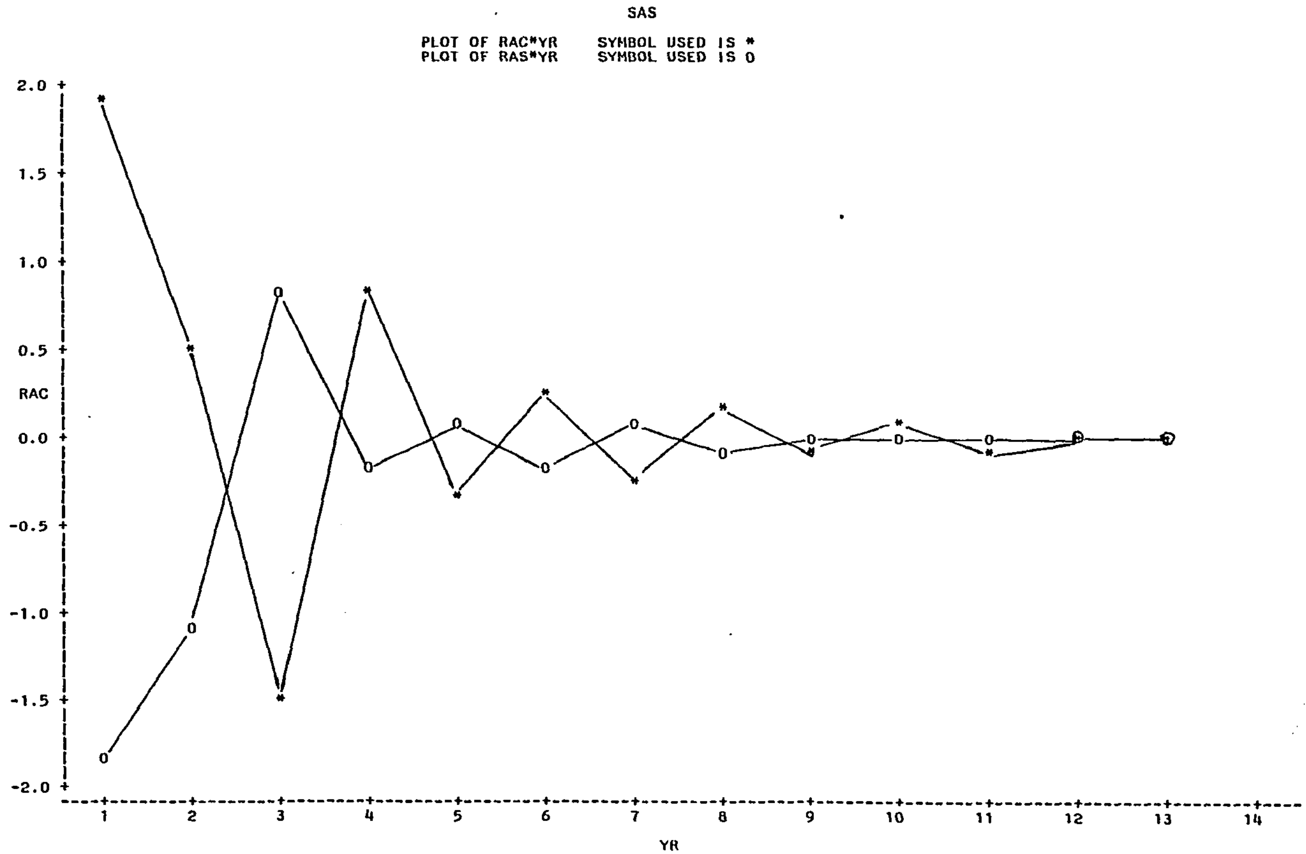

Figure 3.2. Plot of responses of area of corn and area of soybean to one standard deviation shock in yield of corn

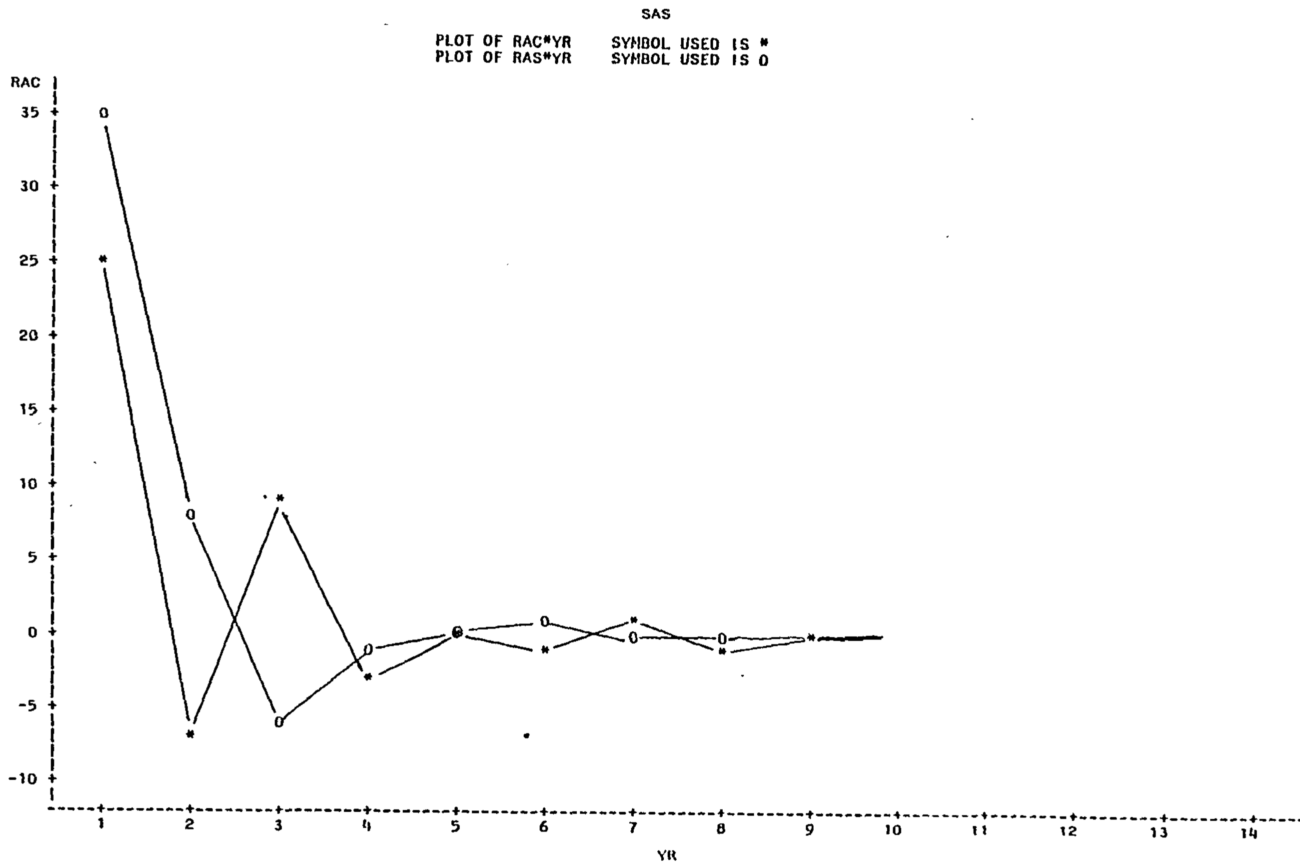

Figure 3.3. Plot of responses of area of corn and area of soybean to one standard deviation shock in area of corn

D). A one-standard-deviation innovation in area of soybeans results in a decrease in relative price of soybean to price of corn in the first period. A one-standard-deviation innovation in area of corn results in an increase in the relative price in the first period. While the negative response of price to an innovation in area of soybean is greater in the first period, the response of price to innovation in area of corn takes a longer period to converge. Furthermore, like the responses of areas to almost all innovations, the response of price to innovations in areas exhibit a cyclical pattern.

Table 3.1 summarizes the results of 12 years ahead decomposition of the forecast error variance. About 55% of the forecast error variance in prices is accounted for by innovations in prices. This result also provides some support to our assumption of price exogeneity. In the long run, yields seem to account for more of the forecast error variance in land allocation than prices while a greater percentage is accounted by prices in the short-run (Appendix D). This result, of course, is for this particular ordering.

Some characteristics of these results are particularly important. First, there is some dynamic interaction among output (land allocation), yields and relative prices. In particular, the data show that innovation cause alternating

Table 3.1. Percentage of forecast error variance 12 years ahead produced by each triangularized innovation[a]

	Innovation				
	AS	AC	YS	YC	P
AS	24	14	21	17	19
AC	10	28	15	36	10
YS	3	8	79	8	1
YC	5	20	12	56	6
P	23	17	2	3	55

[a]Detailed results are given in Appendix D.

positive and negative influences on land allocation. Responses of price show similar pattern to innovations in areas. Even though responses of areas to innovations in yields are, in general, smaller in absolute value than to innovations in prices, the responses to innovations in yields persist over a longer period. The responses of price to innovations in area of corn is small (compared to innovations in area of soybeans) but persistent over a longer period. Second, the data seem to be consistent with the assumption that prices are not Granger-caused by output and yields. Third, in the long-run, innovations from yield account for a larger percentage of the forecast error variance in land allocation than do innovations from prices.

Stability Over Time

One assumption underlying classical linear model is that the econometric structure generating the sample observations remains unchanged over all observations. This assumption includes a single parameter vector relating the dependent variable and the independent variables, a single set of error process parameters and a single functional form. A frequent concern is that the parameters change over time or as the sample size increases.

In the land allocation model of Chapter II, an implicit assumption was made that structural change in the production function did not occur over the sample period. This assumption will be tested here. Tests for stability of the equations for areas, yields and relative price, and other variables are performed.

One way of testing for structural change is to partition the sample into two (or more) groups and then perform a test of the null hypothesis that parameters of these groups are equal. This approach is feasible when a large sample exists, and a criteria is available for partitioning the sample. Alternatively, one can use dummy variables to account for the expected change. This can be done by adding dummy variables to that part of the sample where changes are expected to have occurred. A test of the null hypothesis that the fit of the model with and without the dummy variables is the same is then performed.

For the five-variable vector of areas, yields and relative price we do not have a priori reason to believe that a structural change has taken place in any given year. Therefore, the sample was split arbitrarily into three parts:

(1) At 1958, i.e., structural shifts have taken place after the first 11 years;

(2) At 1959 and 1969, i.e., the middle 11 years of the sample period are different from the first and the last 11 years; and

(3) At 1969, i.e., there has been structural change during the last third of the sample period.

A set of regressions was run by adding a set of dummy variables (for the smaller segment of the sample) to the right-hand-side of all regressions in the system, accounting for the period being tested. The test statistic for the VAR of AS, AC, P, YS and YC is the likelihood ratio statistic which is calculated as described in Equation 3.10a-3.10b, comparing the fit of the system with and without dummy variables. The test statistic for each single equation in the system is the usual F-statistic. Tables 3.2, 3.3 and 3.4 present the results.

The tabular value at .05 of $F(1,8)$ is 5.32 and of $\chi^2(5)$ is 11.07. In all the three cases, the sample value of the χ^2 is greater than the critical value of the χ^2. Thus, the hypothesis of no structural change is rejected and the data supports the hypothesis that some structural change has taken place. On the other hand, equation by equation application of the test of the hypothesis of no structural change cannot be rejected for all except the equation for yield of soybeans (YS). As Sims (1980) pointed out, the likelihood test can be biased: ". . . the statistics (the likelihood statistics) are probably biased against the null hypothesis when the degrees of freedom in the test statistic are small". Thus, the conclusions of the overall test may be too strong. The analysis proceeds under the assumption of no structural change.

The stability of the production function is tested next. From Equation 2.18 of Chapter II, average output per unit of land can be written as:

$$y_t = d_0 - \frac{d_1}{2} A_{1t} + d_1(\bar{A}_t - A_{1t-1}) + a_t \qquad (3.8)$$

$$a_t = \rho a_{t-1} + U_t^a$$

Table 3.2. Test for model homogeneity: 1948-1959 vs. 1959-1980

Equation	Test statistic
AS	F(1,8) = .11
AC	= .12
YS	= .19
YC	= 1.37
P	= .66
Overall	$\chi^2(5)$ = 11.22

Table 3.3. Test for model homogeneity: 1959-1969 vs. 1948-1958 and 1970-1980

Equation	Test statistic
AS	F(1,8) = 1.87
AC	= .85
YS	= 6.29
YC	= 2.26
P	= 1.18
Overall	$\chi^2(5)$ = 15.62

Table 3.4. Test for model homogeneity: 1948-1969 vs. 1970-1980

Equation	Test statistic
AS	F(1,8) = 1.14
AC	= .105
YS	= 5.89
YC	= 3.58
P	= 1.63
Overall	$\chi^2(5)$ = 13.11

where

U_t^a's are iid; and

y_t is yield, bushels of corn per acre of land.

Equation (3.8) seems to have the properties of a regression equation with serially correlated disturbances and no lagged dependent variable on the right-hand-side. However, since a_t and A_{1t} are correlated, the orthogonality condition or conditional expectation $E[a_t/\bar{A}_t, A_{1t}, A_{1t-1}] = 0$ is violated. Therefore, the generalized least squares estimation is not a consistent estimator.

Applying the operator $(1-\rho L)$ to (3.8) and writing the result in terms of deviations from means, a new specification for y_t is obtained:

$$y_t = \rho y_{t-1} - \frac{d_1}{2} A_{1t} + \rho \frac{d_1}{2} A_{1t-1} + d_2(\bar{A}_t - A_{1t-1})$$

$$+ \rho d_2(\bar{A}_{t-1} - A_{1t-2}) + U_t^a \qquad (3.9)$$

The conditional expected value of U_t^a in Equation (3.9) is zero, i.e.,

$$E[U_t^a/y_{t-1}, A_{1t}, A_{1t-1}, A_{1t-2}, \bar{A}_t, \bar{A}_{t-1}] = 0.$$

Equation (3.9) can now be estimated using nonlinear regression methods.

To test the stability of the parameters of this production function over time, a procedure suggested by Judge et al. (1980) is applied. Consider the model $y_t = X_t\beta + e_t$. A test for discrete shifts in slope parameters[7] when the shift point is unknown has been suggested by Farley, Hinich, and McGuire (1975).[8] A discrete shift in the slope at an unknown point can be approximated by a continuous linear shift using a time varying parameters model

$$y_t = X_t\beta_t + e_t \tag{3.10}$$

where

$$\beta_t = \beta + t\delta.$$

This means that the "full" model can be written as

$$y_t = X_t\beta + X_t t\delta + e_t \tag{3.11}$$

Thus, the full model is obtained by replacing ρ, d_1, and d_2 by $\rho + t\delta_1$, $d_1 + t\delta_2$, and $d_2 + t\delta_3$ in (3.9). The test of the null hypothesis of no slope change is the likelihood ratio test of the hypothesis that $\delta=0$, i.e., $\delta_1 = d_2 = \delta_2 = \delta_3 = 0$. Equation (3.9) is then the "reduced" or restricted model. The "full" and "reduced" models were estimated using the Gauss-Newton method. The results are presented in Table 3.5.

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[7]The model of Chapter II which includes the production function will be estimated in terms of deviations from means and linear trends. Therefore, we only need to test shifts in slope parameters of the production function.

[8]We do not have information after which year, if any, structural shifts have occurred. Therefore, we let the parameter change on the basis of time.

Table 3.5. Estimated parameters of the production function

Model	ρ (SEE)	d_1 (SEE)	d_2 (SEE)	δ_1 (SEE)	δ_2 (SEE)	δ_3 (SEE)	Residual sum of squares
Full	.27 (.224)	.0088 (.00073)	.0044 (.00031)	.0039 (.0023)	.00072 (.00049)	.00033 (.00020)	1053.64
Reduced	.39 (.175)	.00023 (.00016)	.00023 (.000087)				1208.59

The sample value of the test statistic for the null hypothesis of no change in the parameters of the production function is $F(3,25) = 1.23$. The tabular value at .05 of $F(3,25)$ is 2.99. Thus, we fail to reject the null hypothesis of no changes in the parameters of the production function. This conclusion is clearly conditioned by the particular specifications of the parameter change that is stated in Equation 3.10.

In summary, in this chapter, some of the dynamic interactions among land allocations, yields and relative prices are examined. The stability of the equations for these variables as well as the parameters of the production function are also tested. Even though the test results were inclusive in some cases, the assumption of no structural shifts over the sample period is maintained. The next chapter focuses on a strategy for the joint estimation of the equations for the decision rule for land allocation, the production function and the processes generating the exogenous variables.

CHAPTER IV. DATA AND ESTIMATION

In Chapter II, a dynamic land allocation model for two crops was derived for a representative farmer whose objective function was assumed to be maximization of present discounted value of expected net income. This model gave a set of simultaneous equations that contain within equation and cross equation restrictions. In this chapter, the land allocation model is tested by fitting it to data on land allocated to corn and soybean in Iowa. Data on acreage, production, yield and price for corn and soybeans and on total cultivated land are obtained from the publications of Iowa and United States Department of Agriculture.[1] Using the estimated equation for land allocation, elasticities of supply are computed and historical simulation of corn acreage is made.

This chapter is organized as follows: In Sections 1 and 2, the data and government policies that have been affecting the production and price of corn and soybeans are discussed. In Section 3, the model is specified, estimated and the results presented. The supply elasticities are discussed in Sections 4 and 5, the land allocation model is compared to Nerlove-type models. The simulation results are presented in the last section.

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[1]The data are given in Appendix E.

Corn and Soybeans in Iowa

The selection of corn and soybean acreage as the focus of the empirical analysis is motivated by the major role that these crops play in the agricultural economy of Iowa. Iowa is an agricultural state that specializes in both crop and livestock production. Of the 36 million acres which form the total area of the state, the acreage in farms has remained roughly stable at 34 million acres or 94% of the state since 1950. Despite the fact that row crops accelerate soil erosion, the share of row crops as a percent of total farm land in Iowa has grown from 40% in 1950 to over 60% in 1980 while the share of nonrow crops has fallen from 40% in 1950 to about 16% in 1980. The corn and soybean acreage accounts for over 85% of the cultivated land during this 80 year period. For the years 1979-81, for example, the total acreage allocated for corn and soybean averaged about 87% of the total cultivated land. Furthermore, Iowa has ranked high nationally in production of corn for grain and production of soybeans for beans.

In addition to fertilizer and pesticides application, crop rotation is employed by Iowa farmers to prevent rapid deterioration of soil fertility that occurs when the same crop is planted year after year on the same acreage. Corn and soybeans are part of the crop rotation system that a large share of Iowa farmers follow (see e.g., Heady and Langley, 1981).

Government Policy

For most of the years since the 1930s, American agriculture has been under some sort of supply control or land retirement programs - acreage allotments, price supports, payment in kind etc.[2] These programs have especially affected the production of feed grains.

There were two main components of the feed grain program from 1948-1958: price support and acreage allotments. In most years when acreage allotments were in effect, compliance was a requisite for obtaining a price support for corn. The acreage allotments were in effect in 1950 and 1954-1958. In 1959 and 1960, no allotment was in effect for corn, and United States corn production was exceptionally high and in 1950 exceptionally low (Cochran and Ryan, 1976). In 1961, a new program ("acreage diversion payments") where producers, in order to qualify for price support, were required to divert land from corn and sorghum to conserving uses was introduced. In 1967, some changes in the program were made to relax production restrictions. As a result, production of corn rose sharply in 1967.

[2] See for e.g., Cochrane and Ryan (1976) or A. Essel (1980) for detailed description of these and other programs.

Because of high world demand and other factors, no significant supply control restriction was imposed on corn production during the 1970s.

The government has not had acreage allotment as diversion programs for soybeans. Soybeans differ from other crops in that demand has matched supplies at or above the support rate in almost all years. This has resulted in steady growth of soybean production.

The impact of these government policies on total production of corn and soybean acreage in Iowa is similar to their impact on the production of these crops in the U.S. As Figure B.1 (Appendix B) shows, the corn acreage in Iowa has three peaks - 1959, 1960 and 1967. These peaks correspond to the years when no corn allotment was in effect (1959, 1960) and to the year when the program was relaxed (1967). The corn acreage in Iowa in 1950 was unusually low because of the acreage allotment in that year. The corn acreage began an upward trend in the next years when the programs were not in effect. Since the early 1970s, the Iowa corn acreage has a sharp upward trend. In the early 1970s, feed grain set aside program was initiated. Cropland diversion payments did not require a reduction in acreage planted to any particular crop and price supports loans were not contingent upon compliance with planting restrictions for a given crop. These changes might have led to the expansion of corn acreage at the expense of other feed grains.

Figure B.2 (Appendix) shows a steady growth in the acreage of soybeans. There were no acreage allotment programs for soybean and the support price, in almost all the years, was below the market price. Therefore, soybean acreage has not been affected directly by a government program.

Model Specification and Estimation

The objective is to apply the dynamic decision rule developed in Chapter II to Iowa corn and soybean acreage. The model contains a set of equations for a land allocation rule, production function and for the stochastic processes for relative prices and other related variables. The set of equations is to be estimated simultaneously with constraints imposed. The model is specified and tested in this section.

Specification of the model

Given the definitions of variables in Chapter II and replacing crop 1 by corn and crop 2 by soybean, the farmer's maximization problem is given by Equation 2.20 of Chapter II.

Before proceeding further, nonland costs of production should be discussed. Some production costs are incurred at time t, when input decisions are made (e.g., costs of land preparation). Other costs are incurred at time t+1, harvest time (e.g., costs of variable harvesting). The c_{1t}'s and c_{2t}'s are the discounted per acre costs of corn and soybeans, respectively. Data on production costs by crop were available only for the years 1970 to 1981. The data for these eleven years show that relative per acre production cost (c_{1t}/c_{2t}), has been reasonably constant.[3] Thus, the result may not be very sensitive to excluding c_{1t} and c_{2t} from the maximization problem. The decision problem is restated as maximization of the present discount value of gross rather than net income:

$$\underset{\{A_t\}}{\text{Max}}\; E_0 \sum_{t=0}^{\infty} \beta^t \{(d_0+a_{1t})A_{1t} - \frac{d_1}{2}A_{1t}^2 - d_2\bar{A}_tA_{1t}$$

$$- d_2A_{1t-1}A_{1t} + P_{t+1}\bar{A}_t - P_{t+1}A_{1t}\} \qquad (4.1)$$

From Chapter II, Equation (2.14), optimal land allocation is:

$$A_{1t} = \lambda_1 A_{1t-1} - \frac{\lambda_1}{d_2} \sum_{i=0}^{\infty} (\lambda_1\beta)^i [d_0+E(a_{1t+i}) + d_2E(\bar{A}_{t+i})$$

$$- E(P_{t+1+i})] \qquad (4.2)$$

---

[3]For the eleven years $\frac{c_{2t}}{c_{1t}}$ has been 2.1, 2.3, 2.4, 2.2, 2.3, 2.3, 2.4, 2.6 and 2.7.

where

$$|\lambda_1|<1 \text{ and } \lambda_1^{-1} = -\frac{d_1}{d_2} - \beta\lambda_1$$

$$\frac{d_1}{d_2} > 1+\beta.$$

To estimate the parameters of the model, it is sufficient to fit the model to variables from which the mean and linear trend have been removed.[4] Therefore, all the variables, except corn acreage, have been filtered with a constant term and linear trend. Corn acreage has been filtered with a constant term, linear trend and dummy variables for presence of a land retirement program for corn. Henceforth, no equation contains a constant term. In order to make Equation (4.2) estimable, E(·) must be expressed in terms of observable variables. First, however, a decision must be made on variables to include in the price equation; i.e., variables that contain information about relative prices. Bivariate Granger causality tests (Granger, 1969) are applied to the relative price P_t and a variable X_t; where both X_t and P_t have been adjusted for a linear trend. A pair of regression is needed

[4] If a_0, A_0 and P_0 are the constant terms of the a_{1t}, $\bar{A}_t$ and P processes, respectively, then the estimated constant term in Equation (4.2) is $-\frac{\lambda_1}{d_2}(d_0+a_0+A_0-P_0) \cdot \frac{1}{(1-\lambda_1\beta)}$. Thus, a_0, A_0, P_0 and d_0 are not identifiable.

to complete the tests. They are:

(1) H_0: $X_t \rightarrow P_t$ (X_t "Granger causes" P_t)

$$P_t = a_0 + \sum_{i=1}^{r_1} a_i P_{t-i} + \sum_{i=1}^{k_1} C_i X_{t-i} + e_{1t}$$

The hypothesis is equivalent to testing the null hypothesis that $C_i=0$, $i = 1,\ldots k_1$.

(2) H_0: $P_t \rightarrow X_t$ (P_t "Granger causes" X_t)

$$X_t = b_0 + \sum_{i=1}^{r_2} b_i X_{t-i} + \sum_{i=1}^{k_2} d_i P_{t-i} + e_{2t}$$

This test is equivalent to testing H_0: $d_i=0$ for $i=1,\ldots,k_2$. A variable X_t will be included in the RHS of the price equation, if H_0 is "accepted" in (1) and H_0 is rejected in (2). However, there can be unlimited number of variables for which such tests should be performed. Variables are limited to ones employed in other studies and to variables suggested by economic theory. For the price equation, several variables including government price support for corn and soybean, livestock prices and futures market for these crops were tried. In case of futures prices, there was stronger evidence of causality going from observed market prices to futures prices rather than in the reverse direction. These results are consistent with Choi's (1982) findings.

The following equation met the requirements of the test:

$$P_t = \alpha_1 P_{t-1} + \alpha_2 G_{t-1} + \alpha_2 G_{t-2} + U_t^P \quad |\alpha_1|<1 \qquad (4.3)$$

where

$$P_t = \frac{\text{price of soybean}}{\text{price of corn}} \times \frac{\text{production of soybean}}{\text{acreage of soybean}}$$

G_t = ratio of government price support for corn in dollars per bushel to market price for corn in dollars per bushel

Tests on lag length of G_t supported a lag length of one:

$$G_t = gG_{t-1} + U_t^S, \quad |g|<1. \qquad (4.4)$$

In order to find the expectations of the exogenous variables ($a_{1t}, \bar{A}_t, P_t$) in Equation 4.2, it is necessary to define the processes generating these variables. The process generating P_t is given by Equations (4.3) and (4.4). The total cultivated land, $\bar{A}_t$, is found to follow a second order autoregressive process.

$$\bar{A}_t = \gamma_1 \bar{A}_{t-1} + \gamma_2 \bar{A}_{t-2} + U_t^A \qquad (4.5)$$

As Sargent (1978b) stated:

> . . . optimizing rational expectations models does not entirely eliminate the need for side assumptions not grounded in economic theory. Some arbitrary assumptions about the nature of serial-correlation structure of the disturbance and/or about strict econometric exogeneity are necessary in order to proceed with estimation (Sargent 1978b, p. 479).

It is assumed that the a_{1t} process has a first-order autoregressive representation.

$$a_{1t} = \rho a_{1t-1} + U_t^a, \quad |\rho|<1 \quad \text{and} \quad U_t^a \text{ is iid.} \qquad (4.6)$$

Given the forms (4.3), (4.5) and (4.6) for the stochastic processes P_t, $\bar{A}_t$, and a_{1t} and the prediction formula in Appendix A, the following equations are obtained for the prediction part of (4.2):

$$\frac{\lambda_1}{d_2} \sum_{i=0}^{\infty} (\lambda_1\beta)^i E(a_{1t+i}) = \frac{\lambda_1}{d_2} \cdot \frac{\rho}{(1-\lambda_1\beta\rho)} a_{1t-1} \qquad (4.7)$$

$$\lambda_1 \sum_{i=0}^{\infty} (\lambda_1\beta)^i E(\bar{A}_t) = \frac{\lambda_1}{(1-\gamma_1\lambda_1\beta-\gamma_2(\lambda_1\beta)^2)} \cdot \bar{A}_t + \frac{\lambda_1^2\beta\gamma_2}{(1-\gamma_1\lambda_1\beta-\gamma_2(\lambda_1\beta)^2)} \cdot \bar{A}_{t-1} \qquad (4.8)$$

$$\frac{\lambda_1}{d_2} \sum_{i=0}^{\infty} (\lambda_1\beta)^i E(P_{t+1+i}) = \frac{\lambda_1}{d_2} \cdot \frac{\alpha_1}{(1-\lambda_1\beta\alpha_1)} \cdot P_t + \frac{\lambda_1}{d_2} \frac{(\alpha_2+\lambda_1\beta\alpha_3)}{(1-g\lambda_1\beta)(1-\alpha_1\lambda_1\beta)} G_t + \frac{\lambda_1}{d_2} \cdot \frac{\alpha_3}{(1-\lambda_1\beta\alpha_1)} \cdot G_{t-1} \qquad (4.9)$$

After substituting Equations 4.7-4.9 in 4.2, the final decision rule (4.2) is as follows:

$$A_{1t} = \lambda_1 A_{1t-1} + \Pi_1 \bar{A}_t + \Pi_2 \bar{A}_{t-1} + \Pi_3 P_t + \Pi_4 G_t + \Pi_5 G_{t-1} + \Pi_6 a_{1t-1} \qquad (4.10)$$

where

$$\lambda_1^{-1} = -\frac{d_1}{d_2} - \beta\lambda_1 \quad \text{and} \quad \frac{d_1}{d_2} > 1+\beta$$

$$\Pi_1 = -\frac{\lambda_1}{(1-\gamma_1\lambda_1\beta-\gamma_2(\lambda_1\beta)^2)}$$

$$\Pi_2 = -\frac{\lambda_1^2\beta\gamma_2}{(1-\gamma_1\lambda_1\beta-\gamma_2(\lambda_1\beta)^2)}$$

$$\Pi_3 = \frac{\lambda_1}{d_2} \cdot \frac{\alpha_1}{(1-\alpha_1\lambda_1\beta)}$$

$$\Pi_4 = \frac{\lambda_1}{d_2} \cdot \frac{(\alpha_2+\lambda_1\beta\alpha_3)}{(1-\alpha_1\gamma_1\beta)(1-g\lambda_1\beta)} \qquad (4.11)$$

$$\Pi_5 = \frac{\lambda_1}{d_2} \cdot \frac{\alpha_3}{(1-\alpha_1\lambda_1\beta)}$$

$$\Pi_6 = -\frac{\lambda_1}{d_2} \cdot \frac{\rho}{(1-\lambda_1\beta\rho)}$$

Equation (4.11) is a set of restrictions on the parameters that is implied by the rational expectations hypothesis.

Estimation

To test the dynamic model, the production function, the acreage decision rule and the exogenous processes are expressed as a system of simultaneous equation and are estimated jointly. This approach insures efficient estimation.

Equation (4.10) is deterministic in the sense that all the variables on the RHS are assumed to be known which makes the relationship exact. Although a_{1t-1}, is assumed to be in the farmer's information set, it is not observable to the econometrician. Therefore, operate with $(1-\rho L)$ on Equation (4.10) to obtain

$$\begin{aligned} A_{1t} = {} & (\lambda_1+\rho)A_{1t-1} - \rho\lambda_1 A_{1t-2} + \Pi_1\bar{A}_t + (\Pi_2-\rho\Pi_1)\bar{A}_{t-1} \\ & - \rho\Pi_2\bar{A}_{t-2} + \Pi_3 P_t - \rho\Pi_3 P_{t-1} + \Pi_4 G_t + (\Pi_5-\rho\Pi_4)G_{t-1} \\ & - \rho\Pi_5 G_{t-2} + \Pi_6 U^a_{t-1} \end{aligned} \tag{4.12}$$

which does not contain a_{1t-1}. When Equation (2.18), the production function for corn is divided by A_{1t}, an equation for average yield of corn is obtained:

$$Y_t = \rho Y_{t-1} - \frac{d_1}{2}A_{1t} + (\rho\frac{d_1}{2} - d_2)A_{1t-1} + \rho d_2 A_{1t-2}$$
$$+ d_2\bar{A}_t + \rho d_2\bar{A}_{t-1} + U_t^a \qquad (4.13)$$

Thus, the system of equations to be estimated consist of equations for the decision rule (4.12), average yield derived from the production function (4.13), and for the processes generating the exogenous variables P_t, G_t and $\bar{A}_t$, Equations (4.3), (4.4) and (4.5), respectively. The system of these equations is given in Equation (4.14):

$$A_{1t} = (\lambda_1+\rho)A_{1t-1} - \rho\lambda_1 A_{1t-2} + \Pi_1\bar{A}_t$$
$$+ (\Pi_2-\rho\Pi_1)\bar{A}_{t-1} - \rho\Pi_2\bar{A}_{t-2} + \Pi_3 P_t - \rho\Pi_3 P_{t-1}$$
$$+ \Pi_4 G_t + (\Pi_5-\rho\Pi_5)G_{t-1} - \rho\Pi_5 G_{t-2} + \Pi_6 U_{t-1}^a$$
$$P_t = \alpha_1 P_{t-1} + \alpha_2 G_{t-1} + \alpha_3 G_{t-2} + U_t^p \qquad (4.14)$$
$$\bar{A}_t = \gamma_1\bar{A}_{t-1} + \gamma_2\bar{A}_{t-2} + U_t^A$$
$$G_t = gG_{t-1} + U_t^G$$
$$Y_t = \rho Y_{t-1} - \frac{d_1}{2}A_{1t} + (\rho\frac{d_1}{2} - d_2)A_{1t-1} + \rho d_2 A_{1t-2}$$
$$+ d_2\bar{A}_t - \rho d_2\bar{A}_{t-1} + U_t^a$$

Where the Π_i's (i = 1,...6) are defined in (4.11).

This system of equations is highly nonlinear in the parameters and is subject to cross-equation restrictions. In particular, the coefficients in the decision rule are nonlinear function of the coefficients of the yield equation, the stochastic processes and the discount factor.

The system of Equations 4.14 can be written in vector form as

$$\begin{bmatrix} A_{1t} \\ \bar{A}_t \\ P_t \\ G_t \\ Y_t \end{bmatrix} = \begin{bmatrix} 0 & \Pi_1 & \Pi_3 & \Pi_4 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ -\frac{d_1}{2} & d_2 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} A_{1t} \\ \bar{A}_t \\ P_t \\ G_t \\ Y_t \end{bmatrix} +$$

$$\begin{bmatrix} (\lambda_1+\rho) & (\Pi_2-\rho\Pi_1) & -\rho\Pi_3 & (\Pi_5-\rho\Pi_4) & 0 \\ 0 & \gamma_1 & 0 & 0 & 0 \\ 0 & 0 & \alpha_1 & \alpha_2 & 0 \\ 0 & 0 & 0 & g & 0 \\ (\rho\frac{d_1}{2}-d_2) & -\rho d_2 & 0 & 0 & \rho \end{bmatrix} \begin{bmatrix} A_{1t-1} \\ \bar{A}_t \\ P_{t-1} \\ G_{t-1} \\ Y_{t-1} \end{bmatrix} +$$

$$\begin{bmatrix} -\rho\lambda_1 & -\rho\Pi_2 & 0 & -\rho\Pi_5 & 0 \\ 0 & \gamma_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & \alpha_3 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ d_2 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} A_{1t-2} \\ \bar{A}_{t-2} \\ P_{t-2} \\ G_{t-2} \\ Y_{t-2} \end{bmatrix} + \begin{bmatrix} \Pi_6 U^a_{t-1} \\ U^A_t \\ U^p_t \\ U^G_t \\ U^a_t \end{bmatrix}$$

Where U^a_t, U^p_t, G^G_t and U^A_t are innovations defined in Equation (2.38a-d). Thus, $\varepsilon'_t = (\mu_6 U^a_t U^p_t U^G_t U^a_t)$, is a vector of innovations, and ε_t is assumed to have a multivariate normal distribution with variance-covariance matrix $E(\varepsilon_t \varepsilon'_t) = V$. Estimators of the free parameters:

$$\Theta = \{\rho,\ d_1,\ d_2,\ \gamma_1,\ \gamma_2,\ \alpha_1,\ \alpha_2,\ \alpha_3,\ g\}$$

can be obtained by maximizing the likelihood function for the model with respect to θ; the discount factor is set independently. Let $e_t' = (e_{1t}\ e_{2t}\ e_{3t}\ e_{4t}\ e_{5t})'$ be the sample residual vector for θ, then the log likelihood function for the observations on the residuals over $t = 1,2,\ldots,T$ is (see Bard, 1974, p. 94; Anderson, 1974, p. 45).

$$L(\Theta) = \frac{5}{2}T\cdot\log(2\pi) - \frac{T}{2}\log|V| - \frac{1}{2}\sum_{t=1}^{T} e_t'V^{-1}e_t \quad (4.15)$$

For a given θ, with V unknown, the maximum likelihood estimator of V can be obtained from (Bard, p. 66)

$$\tilde{V}(\Theta) = \frac{1}{T}\sum_{t=1}^{T} e_t(\Theta)\cdot e_t'(\theta) = \frac{1}{T}M(\Theta) \quad (4.16)$$

or the concentrated likelihood function is

$$L(\theta) = \frac{5}{2}\,T[\log(\frac{T}{2\pi})-1] - \frac{T}{2}\log\det.\ M(\theta). \quad (4.17)$$

Equation (4.17) may be maximized with respect to the vector of parameters θ to obtain the maximum likelihood estimators of the parameters.

A null hypothesis about the parameters of the model can be tested by employing the likelihood ratio test statistic. If $L_R(\theta)$ is the value of likelihood function of the restricted model (e.g., the restrictions imposed by the rational expectations hypothesis, Equation (4.11) and $L_u(\theta)$ is the value of the likelihood at its unrestricted maximum, then $-\frac{1}{2}\log\left(\frac{L_R(\theta)}{L_u(\theta)}\right)$ is asymptotically distributed as $\chi^2(q)$ where $q = q_u - q_r$ (q_u is the number of parameters to be estimated in the unrestricted model, and q_r is the number of parameters to be estimated in the restricted model). An approximation of the likelihood ratio is $T\{\log|D_r| - \log|D_u|\}$, where T is the sample size, D_r is the variance-covariance matrix of the restricted model, D_u is the variance-covariance matrix of the unrestricted model. The null hypothesis or the restrictions are rejected for large value of the likelihood ratio.

Results

Data for fitting the model are corn acreage (A_{1t}), relative price of soybean to corn (P_t), total cultivated land ($\bar{A}_t$), yield of corn (Y_t) and ratio of government price support for corn to market price for corn (G_t) (see Appendix E). All variables are in terms of deviations from mean and linear trend. The time period is 1948-1980.

Two versions of the system of Equations (4.14) are estimated using the nonlinear estimation procedure in the SAS/ETS 79.6 version.

(1) The restricted (RES) model - the system of Equations (4.14) subject to the within equation and cross equations restrictions (4.11). In this restricted model, there are nine free parameters to be estimated.

They are $\theta_R = \{\lambda_1, d_2, \rho, \alpha_1, \alpha_2, \alpha_3, \gamma_1, \gamma_2, q\}$; β is fixed at .96, which implies a real fixed rate of return of 4% per year.

(2) The unrestricted (URES) model - this is the system of Equations (4.14) without the restrictions (4.11). This version of the model has fifteen parameters $\theta_u = \{d_1, d_2, \rho, \lambda_1, \alpha_1, \alpha_2, \alpha_3, \gamma_1, \gamma_2, g, \Pi_1, \Pi_2, \Pi_3, \Pi_4, \Pi_5\}$. If the restrictions are correctly specified, the restricted version, RES, will not be significantly different from the unrestricted model.

Nonlinear iterative three-stage least squares (IT3SLS) and joint generalized least squares (seemingly unrelated regression) were tried using the "modified Gauss-Newton" method to obtain parameter estimates of both the restricted and the unrestricted models. In all cases, less than 50 iterations were required to obtain convergence to a maximum. The parameter estimates obtained using iterative three-stages least squares and generalized least squares were identical in signs and very close in magnitude. However, the estimates from the iterative three-stage least squares had, on the average, smaller standard errors. Therefore, the results obtained from iterative three-stage least squares are viewed as being superior.

The parameter estimates of the RES model is given in Table 4.1. Actually, there are ten parameters in this model. But only nine of the parameters are free since λ_1, d_1 and d_2 are related by the equation $\frac{d_1}{d_2} = -(\lambda_1^{-1}+\lambda_1\beta)$. Thus, λ_1 and d_2 are estimated and d_1 is recovered from the relation.[5] The estimated parameters of the URES model are presented in Table 4.2.

Using the approximate value of the likelihood ratio statistic $T(\log|D_r|-\log|D_u|)$, the value of the likelihood ratio is 9.68. The critical value of the χ^2 at the .05 significance level is 12.59. Thus, the RES model is not rejected at this 5% significance level. This may be taken as strong support for the specification of the model in general and to the restrictions imposed by the rational expectations hypothesis in particular.

The estimated parameters of the RES (Table 4.1) satisfy all the regularity conditions imposed on the parameters; i.e., $|\lambda_1|<1$, $|\rho|<1$, $|g|<1$, $|\alpha_1|<1$ and the roots of $|1-\gamma_1 Z-\gamma_2 Z^2| = 0$ lie outside the unit circle. The restrictions on the production function parameters are also satisfied; i.e., $d_1>0$ and $d_2>0$. The signs of all the parameters are as expected. The positive sign of d_2 supports the claim that soybean production causes net soil fertility deterioration. In this model, the loss in yields

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[5]The standard error of d_1 has not yet been calculated.

Table 4.1. Estimated parameters of the RES model[a,b]

Parameter	Estimate
λ_1	-.0479 (.0377)
ρ	.479 (.108)
α_1	.0033 (.0091)
α_2	73.27 (14.96)
α_3	-82.55 (14.42)
γ_1	1.59 (.128)
γ_2	-.666 (.131)
d_2	.0011 (.00093)
g	.564 (.108)
d_1	.0226

[a]Standard errors are given in parentheses. The The determinant of the var-covariance matrix is 8.3228E + 11.

[b]See p. 96 for the model.

Table 4.2. Estimated parameters of the URES model[a]

Parameter	Estimates
λ_1	.033 (.0262)
ρ	.152 (.162)
α_1	-.084 (.0126)
α_2	86.87 (16.25)
α_3	-90.68 (14.76)
γ_1	1.59 (.124)
γ_2	-.656 (.127)
d_1	.014 (.0053)
d_2	-.0011 (.0009)
g	.727 (.125)
Π_1	.456 (.378)
Π_2	-.172 (.347)

[a]Standard errors are given in parentheses. The de-determinant of the var-covariance matrix is 6.7033E + 11. $T\{\log|D_r| - \log|D_u|\} = 7.14$.

Table 4.2 (Continued)

Parameter	Estimates
Π_3	-9.49 (8.04)
Π_4	2852.10 (1411.65)
Π_5	-765.72 (889.33)

due to continuous planting of the same crop on a given plot of land is represented. Although the loss in yield can be reduced by application of fertilizers and perhaps other inputs, this analysis considers only the effect of crop rotation. Hence, the parameters of the production function (d_1 and d_2) may be subject to "omitted variables" bias.

The results of the RES and URES models are summarized in Tables 4.3 and 4.4.

Supply Elasticities

In this section, the results of the last section are employed to calculate the two types of elasticities defined in Chapter II.

(1) Elasticity with respect to an expected output price change. Elasticity of supply with respect to an

Table 4.3. Estimated RES model

$$
\begin{bmatrix} A_{1t} \\ \bar{A}_t \\ P_t \\ G_t \\ Y_t \end{bmatrix} =
\begin{bmatrix} 0 & .0446 & -.144 & -3315.07 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ -.0113 & .0011 & 0 & 0 & 0 \end{bmatrix}
\begin{bmatrix} A_{1t} \\ \bar{A}_t \\ P_t \\ G_t \\ Y_t \end{bmatrix} +
$$

$$
\begin{bmatrix} .431 & -.020 & .069 & -2073.17 & 0 \\ 0 & 1.59 & 0 & 0 & 0 \\ 0 & 0 & .0033 & 73.27 & 0 \\ 0 & 0 & 0 & .564 & 0 \\ .0043 & -.00053 & 0 & 0 & .479 \end{bmatrix}
\begin{bmatrix} A_{1t-1} \\ \bar{A}_{t-1} \\ P_{t-1} \\ G_{t-1} \\ Y_{t-1} \end{bmatrix} +
$$

$$
\begin{bmatrix} .0229 & .00066 & 0 & 1753.66 & 0 \\ 0 & -.666 & 0 & 0 & 0 \\ 0 & 0 & 0 & -82.55 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ .0011 & 0 & 0 & 0 & 0 \end{bmatrix}
\begin{bmatrix} A_{1t-2} \\ \bar{A}_{t-2} \\ P_{t-2} \\ G_{t-2} \\ Y_{t-2} \end{bmatrix}
$$

Table 4.4. Estimated URES model

$$\begin{bmatrix} A_{1t} \\ \bar{A}_t \\ P_t \\ G_t \\ Y_t \end{bmatrix} = \begin{bmatrix} 0 & .456 & -9.49 & 2852.1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ -.007 & -.0011 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} A_{1t} \\ \bar{A}_t \\ P_t \\ G_t \\ Y_t \end{bmatrix} +$$

$$\begin{bmatrix} .185 & -.2413 & -1.44 & 1199.2 & 0 \\ 0 & 1.59 & 0 & 0 & 0 \\ 0 & 0 & -.084 & 86.87 & 0 \\ 0 & 0 & 0 & .727 & 0 \\ -.00004 & -.00167 & 0 & 0 & .152 \end{bmatrix} \begin{bmatrix} A_{1t-1} \\ \bar{A}_{t-1} \\ P_{t-1} \\ G_{t-1} \\ Y_{t-1} \end{bmatrix} +$$

$$\begin{bmatrix} -.005 & -.026 & 0 & 116.39 & 0 \\ 0 & -.655 & 0 & 0 & 0 \\ 0 & 0 & 0 & -90.68 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ -.0016 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} A_{1t-2} \\ \bar{A}_{t-2} \\ P_{t-2} \\ G_{t-2} \\ Y_{t-2} \end{bmatrix}.$$

expected change in prices corresponds to the elasticity of supply obtained from a Nerlove-type supply model. The long run elasticity of supply is calculated as:

$$\bar{\eta}_p = \frac{\lambda_1}{d_2} \cdot \frac{1}{(1-\lambda_1)(1-\lambda_1\beta)} \cdot \frac{\bar{P}}{\bar{A}_1} = -.2153 \qquad (4.18)$$

$(\bar{P}/\bar{A}_1 = .00542)$.

Short run elasticities are calculated as:

$$\eta_p(j+1) = \frac{\lambda_1}{d_2}(\lambda_1\beta)^j \cdot \frac{\bar{P}}{\bar{A}_1} \qquad (4.19)$$

$\eta_p(1) = -.236$

$\eta_p(2) = .0109$

$\eta_p(3) = -.00049$

$\eta_p(r) = .000023$

$\vdots$

$\eta_p(1+k) = \eta_p(1)\cdot(-.46)^k$

A price (ratio) change, expected to occur beyond three years from the current period does not seem to have much impact on the current land allocation.

(2) Elasticity with respect to an unexpected change: As discussed in Chapter II the long run elasticity of supply with respect to an unexpected price change is zero.

Computation of elasticities with respect to an unexpected change in prices requires the estimation of the entire system, but not necessarily the identification of the underlying parameters (parameters of the production function). In general, estimation of the land allocation equation and the price equation enable us to analyze acreage responses of one standard deviation shock in prices. As discussed in the last chapter, finding these responses is equivalent to tracing out the moving average representation of the estimated acreage and price equations. If the interest is to compute elasticities with respect to an unexpected change in prices, the estimated equations of the unrestricted model can be used to compute these elasticities.

To compute the short run elasticity of supply with respect to a one standard deviation change in price, consider the land allocation Equation (4.10). Without loss of generality, suppose the only variable in the RHS of (4.10) is price so that:

$$A_{1t} = \lambda_1 A_{1t-1} + \Pi_3 P_t + U_t^{A_1}$$

suppose

$$\bar{A}_1 = A_{1t-1} = P_{t-1} = \bar{P} = 0$$

$$U_t^p = \sigma_p = 25.48$$

$$U_s^p = 0, \; s \neq t$$

$$U_t^{A_1} = 0, \; \forall t$$

Then, using the results from Table 4.3, the land allocation follows:

$$\hat{A}_{1t+j} = \lambda_1 A_{1t+(j-1)} + \Pi_3 P_{t+j}$$

where

$$\hat{P}_{t+j} = (-.084)^j \cdot (25.48) \qquad j=0,1,2,\ldots$$

$$\hat{A}_{1t} = -241.8$$

$$\hat{A}_{1t+1} = 11.25$$

$$\hat{A}_{1t+2} = -1.34$$

$$\hat{A}_{1t+2} = .0993$$

$$\hat{A}_{1t+4} = -.00904$$

Figure 4.1 shows the response of land allocation to a one-standard deviation shock in prices. The short run elasticity is $\gamma(0) = -.0514$. It might be interesting to compare the responses of acreage to unexpected change (shock) in prices with unexpected change in government support prices. To this end, we consider the land allocation Equation (4.10) with all the variables, other than P_t and G_t, ignored. A one standard deviation unit shock is G_t at time t causes an increase of 425,800 acres in acreage of corn. A one-standard deviation shock in prices leads to a decrease of 241,800 acres of corn. In terms of elasticities, the response of corn acreage to a one standard deviation unit shock in G_t is equal to .063.

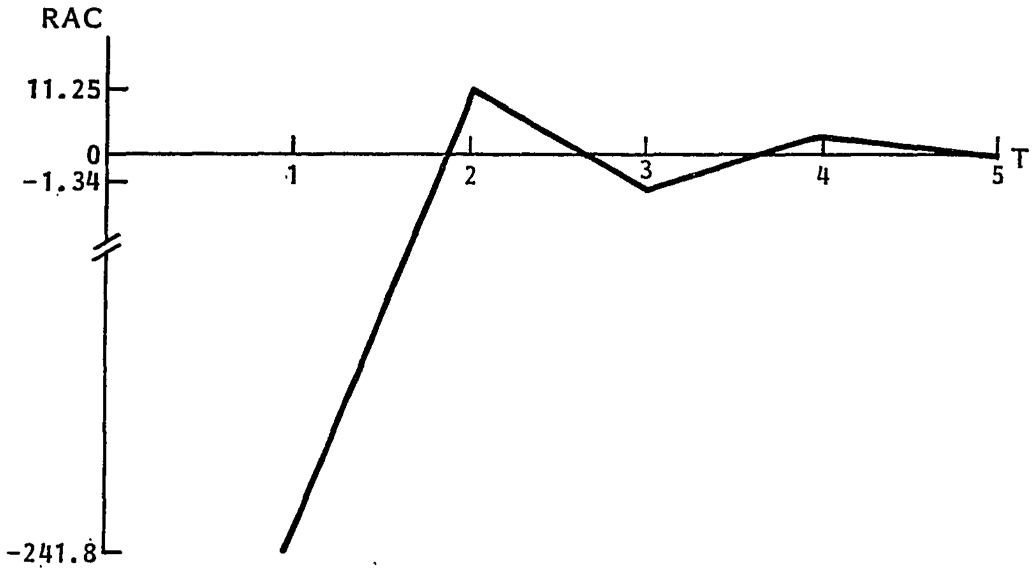

Figure 4.1. Responses of land allocation for corn to a one standard deviation shock in price

This implies that corn area is more responsive to the government support price than to the market prices. This result is consistent with the findings of Houck and Ryan (1972). Houck and Ryan estimated corn acreage supply function for United States by regressing corn area on lagged prices and lagged government support prices and concluded that variations in the weighted support price variable corn acreage better than the lagged market price.

Rational Expectation vs. Nerlove-Type Models

In what follows, the traditional Nerlove-type supply functions and the elasticities derived from these models are compared to the land (acreage) allocation model specified and estimated in this chapter. The land allocation Equation (4.10) is observationally equivalent to Nerlove-type supply models. A typical Nerlove-type supply model has the following equations (see Behrman, 1968, for e.g.):

$$A_t^D = a_0 + a_1 P_{t+1}^e + a_2 Z_t + U_t \qquad (4.20)$$

$$A_t = A_{t-1} + \alpha(A_t^D - A_{t-1}) \qquad (4.21)$$

$$P_{t+1}^e = P_t^e + \beta(P_t - P_t^e) \qquad (4.22)$$

where

A_t = actual area under cultivation at time t

A_t^D = desired area to be cultivation at time t

P_{t+1}^e = price that is expected to prevail at time t+1

P_t = actual price at time t

Z_t = other exogenous variable(s) that affect supply at time t

U_t = disturbance term

β and α are expectation and adjustment parameters, respectively.

P_{t+1}^e and A_t^D are not observable. Solving for P_{t+1}^e in (4.22) in terms of past observed prices and using (4.20), we can write (4.21) as

$$A_t = (1-\alpha)A_{t-1} + \alpha a_0 + \alpha a_1 \beta \sum_{i=0}^{\infty} (1-\beta)^i P_{t-i} + \alpha a_2 Z_t + \alpha U_t. \quad (4.23)$$

To eliminate the infinite sum, multiply the one period lagged value of (4.23) by (1-β) and subtract the result from (4.23) to obtain

$$A_t = b_0 + b_1 A_{t-1} + b_2 A_{t-2} + b_3 P_t + b_4 Z_t + b_5 Z_{t-1} + e_t \quad (4.24)$$

where

$b_0 = \beta\alpha a_0$

$b_1 = (1-\beta) + (1-\alpha)$

$b_2 = -(1-\beta)(1-\alpha)$

$$
\begin{aligned}
b_3 &= \alpha a_1 \beta \\
b_4 &= \alpha a_2 \qquad\qquad (4.25) \\
b_5 &= -(1-\beta)(\alpha a_2) \\
e_t &= \alpha[U_t-(1-\beta)U_{t-1}]
\end{aligned}
$$

Equation (4.24) is equivalent to Equation (2.39) of Chapter II if Z_t in (4.24) is a vector consisting of $\bar{A}_t$ and G_t of Equation (2.39). However, the interpretations of the coefficients are quite different. The elasticities computed from Nerlove-type supply models depend on the serial correlations between output (land allocation) and prices only (b_3). Elasticities computed from models like the one presented in Chapter II depend not only on the serial correlation between output and prices but also on the parameters of the stochastic process governing prices and other exogenous variables and the parameter of the production function.

For the Nerlovian model, the immediate effect of a change in relative price on land allocation (short run) is given by b_3. Hence, the short run elasticity of output (land allocation) with respect to prices, measured at the sample means, is given by

$$\eta'_p = b_3 \frac{\bar{P}}{\bar{A}_{1t}}$$

$$= \alpha\beta a_1 \frac{\bar{P}}{\bar{A}_{1t}} \qquad (4.27)$$

whereas, for the rational expectations model, the immediate short run elasticity is computed as

$$\eta_p = \frac{\lambda}{d_2} \cdot \frac{\bar{P}}{\bar{A}_{1t}} \qquad (4.28)$$

In order to estimate the long-term elasticity, it is necessary to rewrite Equation (4.24) after full adjustment in land allocation has taken place as

$$A^*_{1t} = \frac{1}{1-b_1-b_2} \cdot [b_0 + b_3P_t + b_4Z_t + b_5Z_{t-1} + e_t] \qquad (4.29)$$

Then, the long-run elasticity is given by

$$\bar{\eta}' = \frac{b_3}{1-b_1-b_2} \cdot \frac{\bar{P}_t}{\bar{A}_{1t}}$$

$$= a_1 \cdot \frac{\bar{P}_t}{\bar{A}_{1t}} \qquad (4.30)$$

Thus, the long-run acreage elasticity of the Nerlovian model depends on the correlation between output and prices only. The long-run (expected) elasticity with respect to a change in relative price for the rational expectation model, is given by

$$\bar{\eta}_p = \frac{\lambda_1}{d_2} \cdot \frac{1}{(1-\lambda_1)(1-\lambda_1\beta)} \cdot \frac{\bar{P}}{\bar{A}_{1t}} \tag{4.31}$$

This elasticity is tied to the production function parameters.

Simulation

One way to evaluate the performance of a model is to perform an historical simulation and examine how closely the actual data on a variable is tracked by the predicted values of the variable. In what follows, we present the simulation result of the model estimated in this chapter.

Figure 4.2 presents plots of the predicted values of acreage of corn (PAC) from the dynamic simulation of the restricted model and of the actual corn acreage (AC) for the years 1950 to 1980. The model simulates the turning points reasonably well.

The following criteria are frequently applied to evaluate the performance of a simulation model (Pindyck and Rubinfeld, 1981):

1. Root-mean-square (rms) simulation errot. The rms simulation error for the variable y_t is defined as

$$\text{rms error} = \sqrt{\frac{1}{T}\sum_{t=1}^{T}(y_t^s - y_t^a)^2} \tag{4.32}$$

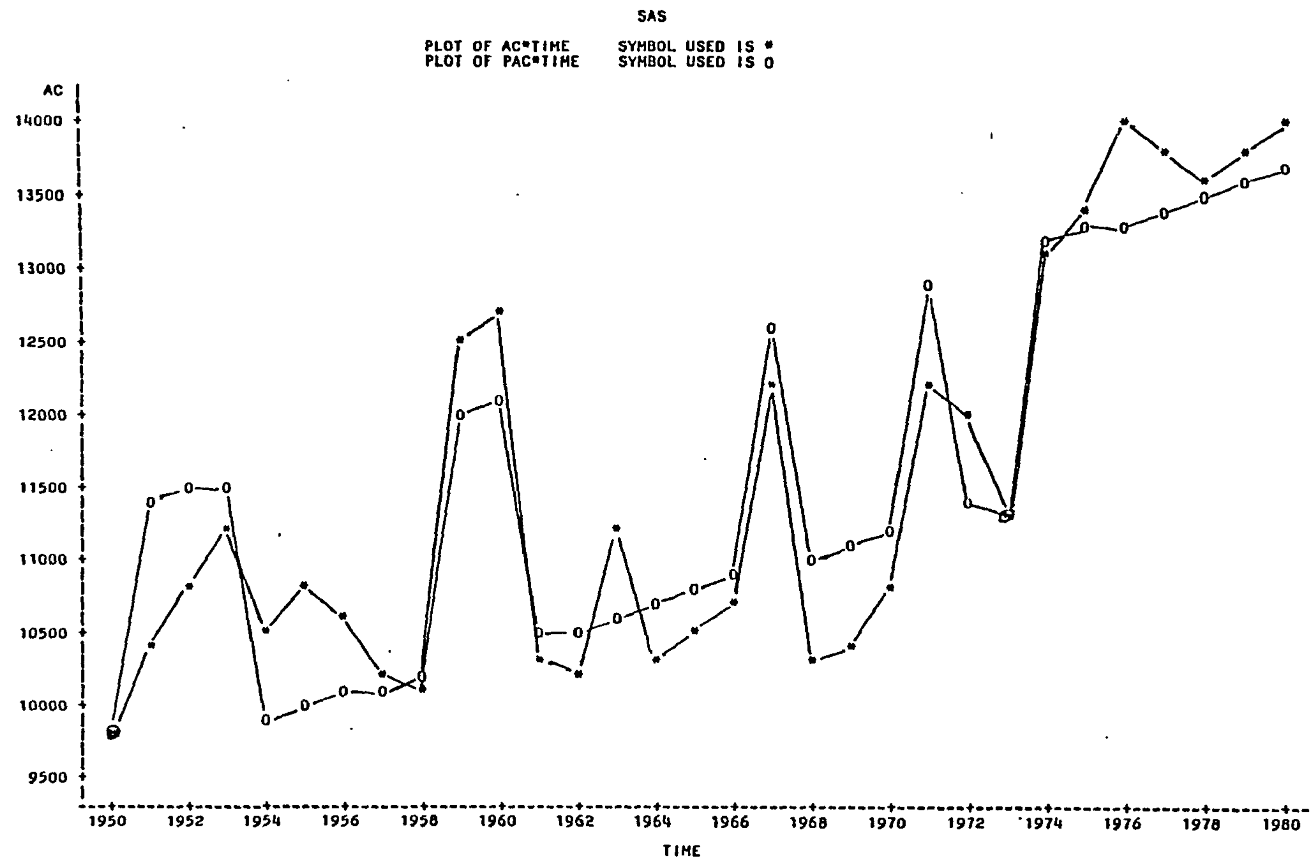

Figure 4.2. Historical simulation of corn acreage: Time bounds: 1950-1980

where

y_t^s = simulated value of y_t

y_t^a = actual value of y_t

T = number of periods in the simulation.

The magnitude of this error is evaluated relative to the mean of y_t^a.

2. Rms percent error: This is defined as

$$\text{Rms \% error} = \sqrt{\frac{1}{T}\sum_{t=1}^{T}\left[\frac{(y_t^s - y_t^a)}{y_t^a}\right]^2} \quad (4.33)$$

The rms % error measures the deviation of the simulated variable from its actual time path in percentage terms.

3. Theil's inequality coefficient: This coefficient is defined as

$$U = \frac{\sqrt{\frac{1}{T}\sum_{t=1}^{T}(y_t^s - y_t^a)^2}}{\sqrt{\frac{1}{T}\sum_{t=1}^{T}(y_t^s)^2} + \sqrt{\frac{1}{T}\sum_{t=1}^{T}(y_t^a)^2}} \quad (4.34)$$

Notice that the numerator of U is the rms error and that U lies between 0 and 1. If U=0, $y_t^s = y_t^a$ for all t and there is a perfect fit. On the other hand, U=1 implies the other extreme. It can be shown that

$$\frac{1}{T}\sum_{t=1}^{T}(y_t^s - y_t^a)^2 = (\overline{Y}^s - \overline{Y}^a)^2 + (\sigma_s - \sigma_a)^2 + 2(1-\rho)\sigma_s\sigma_a \quad (4.35)$$

where $\overline{Y}^s$, $\overline{Y}^a$, σ_s, and σ_a are means and the standard deviations of the series y_t^s and y_t^a, respectively, and ρ is their correlation coefficient.

Divide both sides of (4.6.4) by $\frac{1}{T}\sum_{t=1}^{T}(y_s^t-y_a^t)^2$ (4.36)

to obtain

$$1 = U^m + U^s + U^e$$

where

$$U^m = \frac{(\overline{Y}^s-\overline{Y}^a)^2}{\frac{1}{T}\sum_{t=1}^{T}(y_t^s-y_t^a)^2}$$

$$U^s = \frac{(\sigma_s-\sigma_a)^2}{\frac{1}{T}\sum_{t=1}^{T}(y_t^s-y_t^a)^2}$$

$$U^c = \frac{2(1-\rho)\sigma_s\sigma_a}{\frac{1}{T}\sum_{t=1}^{T}(y_t^s-y_t^a)^2}$$

U^M, U^s and U^c are called the <u>bias</u>, the <u>variance</u>, and the <u>covariance</u> proportions, respectively.

This decomposition of Theil's inequality is a useful means of breaking the simulation error into its characteristic sources. U^m measures the extent to which the average values of the actual and the stimulated series deviate from each other; hence, it is an indication of systematic error. A small value of U^m is a desirable property of a model.

The variance proportion, U^s, indicate the model's ability to capture the degree of variability in the variable of interest. If U^s is large, it means that the actual series has fluctuated considerably while the simulated series shows relatively little fluctuation, or vice versa. Whatever the value of the inequality coefficient U, U^s should be close to zero.

Finally, the covariance proportion, U^c, measures the residual. That is, it represents the remaining error after deviations from average value and average variability have been taken into account. The ideal values for the components of the Theil's U coefficient are $U^m = U^s = 0$ and $U^c = 1$.

Tables 4.5 and 4.6 report the values of these measures.

Table 4.5. rms error and rms % error of historical simulation

Variable	Means	rms Error	rms % Error
A	11518.33	482.37	.99
Y	79.44	6.74	1.45
P	62.47	16.96	1.00
$\bar{A}$	21913.33	1477.93	.90
G	1.04	.184	.96

Table 4.6. Theil's forecast error measures

Variable	U^m	U^s	U^c
A	.01	.03	.96
Y	.02	.40	.58
P	.01	.03	.96
$\bar{A}$	.19	.04	.78
G	.20	.00	.80

For a model that was not designed for forecasting, the model seems to simulate rather well. Among the set of five equations, the equation for yield has the poorest performance. The equation for yield (Y) has the highest rms percentage error and the variance component (U^s) is large (Tables 4.5 and 4.6). The yield equation was derived from the production function and it has some deficiencies. Thus, we are not too surprised that the yield equation does not perform as well as the other equations.

Some highlights of this chapter follow. The model developed in Chapter II was not rejected when fitted to data obtained from the agricultural sector of Iowa. In particular, the "acceptance" of the reduced model gives strong support to the application of rational expectations to farmer's production decisions. The signs of all the parameters are correct; the stationarity conditions imposed on the processes of the exogenous variables and the boundary conditions of the parameters are all satisfied. Also, the computed price elasticities are of reasonable magnitude. The positive results suggest that further improvements and refinements of the model are promising sources of information on agricultural supply functions.

CHAPTER V. CONCLUSIONS

The objective this study was to develop and fit an agricultural supply model for a stochastic and dynamic environment. The work is best viewed as an attempt to suggest a method for analyzing the determinants of the dynamics of agricultural supply. The attempt has a microeconomic foundation of discounted profit maximization of a representative farmer. The empirical analysis is, however, macro because the data are aggregate time series for one state. The empirical analysis combines time series analysis with standard econometric techniques.

Agricultural supply decisions are characterized by the fact that input decisions have to be made before output prices are known. These decisions are made on the basis of, among other things, expected prices. Changes in relative prices are the main constituent of the deriving forces of agricultural supply. The understanding of how farmers form their expectations of future prices (and other variables) is a crucial step in modeling the dynamics of agricultural supply.

Unlike the traditional supply models which assume that farmers expectations of prices are naive (static expectations) or are some weighted average of past prices alone (adaptive expectations), this study promotes the view that farmers take into account past prices and other available information when they make price forecasts (rational expectations).

On the proposition that people base their expectations on past prices alone, Tobin has commented that these "are almost surely inaccurate gauges of expectations. Consumers, workers, and businessmen . . . do read newspapers and they do know better than to base price expectations on simple extrapolation of price series alone" (Tobin, 1972, p. 14). According to rational expectations, price movements that are uncorrelated do not give any information about the future course of prices. That is, if prices are uncorrelated, a rational person would not base his forecasts of future prices on past observed prices. "Price movements observed and experienced do not necessarily convey information on the basis of which a rational man should alter his view of the future. When a blight destroys half the midwestern corn crop and corn prices subsequently rise, the information conveyed is that blight raise prices. No trader or farmer under circumstances would change his view of the future of corn prices, much less of their rate of change, unless he is led to reconsider his estimate of the likelihood of blights" (Tobin, 1972, p. 14).

The cyclical movements in crop production may be an outcome of optimizing behavior of farmers in a dynamic environment rather than due to their alleged backwardness in forming expectations. The results of this study support the optimizing behavior perspective.

By assuming that farmers form rational expectations of prices and other unknown variables in making input decisions and that they maximize the expected present value of their income stream, a land allocation or acreage decision rule was derived. This land allocation decision rule is observationally equivalent to the traditional supply model of Nerlove. However, the two models have different interpretations. In particular, the price elasticities computed from the traditional supply models depend on the correlation between acreage and prices only. The elasticities computed from the model developed in this study depend on the correlation between acreage and prices, the parameters of the objective function and the parameters of the production function.

If the production function and the objective function are correctly specified, the method employed in this dissertation can give a more reliable estimate of the supply elasticity than a traditional supply analysis. The method has a major advantage in policy analysis because policy variables are incorporated into the model. Governmental policy variables are incorporated into the objective function (e.g., subsidy to farmers) or in the stochastic process for prices (e.g., government price supports). When policy variables are included in the model, the price elasticities are functions of, among other things, the parameters of the policy variables. This facilitates making a correct assessment of the effect of a change in governmental policy an agricultural supply.

For the particular model employed in this study, the results obtained are encouraging. The constraints on the parameters are satisfied and the signs of all the parameters are correct. Furthermore, the restrictions implied by the rational expectations hypothesis are supported by the data. The land allocation or acreage decision rule simulates the Iowa corn acreage rather well. However, the estimated yield equation which was derived from the production function simulates relatively poorly.

The elasticities computed from the model are of reasonable magnitude. Nerlove (1958) computed long run elasticities of supply of corn for the U.S. for the period 1903-1932. He obtained estimates ranged from .09 to .35, depending on the assumption he made about the magnitude of the expectation parameter, β. The model of Chapter III gave a long run price elasticity of -.22 for corn[1]. The results also showed that corn area is more responsive to the government support price for corn than to the market price. This conclusion supports Houck and Ryan's (1972) results.

The main disadvantage of the model is the somewhat restricted nature of the production function. A complicated production function does not have a closed-form solution for the maximization problem. Thus, a trade-off exists between accuracy and estimability. In any case, the model in its present form is able to capture the main dynamics and to show good potential as a model for explaining Iowa land allocation.

A note on rational expectations models and some shortfalls of our model follow. Shiller (1978) gives a good critical review of dynamic rational expectation models. In general, rational expectations models give rise to some within equation and cross-equation restrictions on parameters. More often, these restrictions are very highly nonlinear and create multicollinearity among the parameters and results in the ultimate "rejection" of the restricted model (e.g., Langley, 1982). One way

[1]The elasticity is negative since we have the price of corn in the denominator of the price ratio on which the elasticity is computed. Note that while Nerlove used national data, ours is Iowa data. Also, the sample period is quite different.

to go about this problem is to use the "quasirational expectation" approach as suggested by Nerlove and others (Nerlove et al., 1979).

If agents form rational expectations of unknown variables, they must know the actual probability distribution of these variables and use them to form forecasts. This is a strong assumption. According to McCallum (1980, p. 38), there are two common criticisms. First, it may be unrealistic to assume that agents use all information that is available. Second, it may be unrealistic to assume that agents use information as intelligently as the hypothesis claims. Fisher (1982), however, argues that as in any economic model, these assumptions are made so that approximate solutions to some practical problems can be obtained.

Another criticism is the simplifying assumptions that information is costless and agents learn instantly. Some attempts have been made to incorporate a learning process into the rational expectations models (e.g., De Canio, 1979) and costs of acquiring information (Feige and Pearce, 1976). These early attempts are unsatisfactory.

In the particular model of Chapter II, only one input (land) was considered. Further research should include the role of other inputs such as fertilizer and capital. If the production function is separable among inputs, the decision rule derived in this study holds with some minor modifications. In this study, only one aspect of the technology, the deterioration of land fertility or soil erosion, has been considered. For our case, this aspect seems to be able to capture the main phenomenon of crop rotation and the resulting fluctuation in land allocation. Future research should incorporate other important elements of the crop production technology.

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ACKNOWLEDGMENTS

The content of this dissertation reflects intellectual interaction with a number of individuals at Iowa State University. I owe special thanks to one of my major professors, Dr. Wallace E. Huffman for his guidance, close supervision and detailed suggestions both in shaping my methodological approach and in organizing my written work. I also thank my other major professor, Dr. John A. Miranowski, for his helpful comments in the various drafts of this dissertation. His comments were of considerable help.

I have benefitted a lot from the number of discussion sessions I had with Dr. Barry L. Falk. His help in some of the mathematical derivations is greatly appreciated.

I am grateful to the other members of my committee, Dr. Wayne A. Fuller, Dr. Dennis R. Starleaf and Dr. James A. Stephenson. Acknowledgment is due to the Department of Economics and the Agricultural Experiment Station, Iowa State University and the Ford Foundation for the financial support.

Finally, I would like to express my appreciation to Dr. John F. Timmons for his help in my graduate study and Mrs. Denise Cropp whose help was a necessary condition for the completion of this dissertation.

APPENDIX A: OPTIMAL PREDICTIONS

The optimal land allocation is

$$A_{1t} = \lambda_1 A_{1t-1} - \frac{\lambda_1}{(d_2)} \sum_{i=0}^{\infty} (\lambda_1\beta)^i [d_0 + E(c_{2t+i}) - E(c_{1t+i})$$

$$+ E(a_{1t+i}) + d_2 E(\bar{A}_{t+i}) - E(P_{t+1+i})]$$

In order to get an estimable equation for A_{1t}, expression for the expectation operator must be derived. In particular, we need formulas for $\sum_{i=0}^{\infty} \lambda^i E(a_{1t+i})$ and $\sum_{i=0}^{\infty} \lambda^i E(P_{t+i+1})$ while $\lambda = \lambda_1\beta$.

Note that since $|\lambda| < 1$, the infinite sum is convergent so that we need not consider the constant terms.

Following a long tradition in these problems, assume that the law of motion governing the series P_t and a_{1t} are known by the decision-maker. Let the stochastic processes be:

$$a_{1t} = \rho_1 a_{1t-1} + \rho_2 a_{1t-2} + \ldots + \rho_q a_{1t-q} + U_t^a$$

or

$$\rho(L) a_{1t} = U_t^a \tag{A.1}$$

where

$$\rho(L) = 1 - \rho_1 L - \ldots - \rho_q L^q$$

We assume P_t is the first element of a vector autoregressive process that satisfies:

$$\delta(L) X_t = U_t^x \tag{A.2}$$

where X_t and U_t^x are each (pxl) vector; and

$$\delta(L) = I - \delta_1 L - \ldots - \delta_r L^t.$$

(U_t^a, U_t^x) are innovations for the joint $(a_{1t} x_t)$ process. It follows that

$$E(U_t^a | \Omega_{t-1}) = 0, \text{ and}$$

$$E(U_t^x | \Omega_{t-1}) = 0, \text{ where}$$

Ω_{t-1} is as defined in the text.

We further assume that a_{1t} and X_t are jointly covariance stationary. We want formulas for the terms:

$$\ell \sum_{i=0}^{\infty} \lambda^i E_t X_{t+i} = \sum_{i=0}^{\infty} \lambda^i E_t P_{t+i}, \text{ and } \sum_{i=0}^{\infty} \lambda^i E_t a_{1t+i}$$

where $\ell = (1,0,0,\ldots,0)$ is a (lxp) row vector.

By stationarity of a_{1t} and X_t, we can write the moving average representation:

$$a_{1t} = \rho(L)^{-1} U_t^a = \phi(L) U_t^a = [\sum_{j=0}^{\infty} \phi_j L^j] U_t^a \tag{A.3}$$

$$X_t = \delta(L)^{-1} U_t^x = \psi(L) U_t^x = [\sum_{i=0}^{\infty} \psi_j L^j] U_t^x \tag{A.4}$$

The Wiener-Kolmogorov prediction formulas is

$E_t X_{t+1} = [\frac{\psi(L)}{L^i}]_+ U_t^x$, while $[\ ]_+$ is the annihilation operator (see Sargent, 1981). Therefore,

$$E_tX_{t+i} = [\sum_{j=i}^{\infty} \psi_j L^{j-i}]U_t^x \tag{A.5}$$

Then, we have,

$$\sum_{i=0}^{\infty} \lambda^i E_tX_{t+i} = [\sum_{i=0}^{\infty} \lambda^i \sum_{j=i}^{\infty} \psi_j L^{j-1}]U_t^x$$

$$= \gamma(L)U_t^x \tag{A.6}$$

where $\gamma(L) = \sum_{i=0}^{\infty} \sum_{j=i}^{\infty} \lambda^i \psi_j L^j L^{-i}$

Interchanging of summations gives

$$\gamma(L) = \sum_{j=0}^{\infty} \sum_{i=0}^{\infty} \lambda^i \psi_j L^j L^{-i}$$

$\gamma(L)$ can now be rewritten as (see Hansen and Sargent, 1981):

$$\gamma(L) = \frac{[\sum_{j=0}^{\infty} \psi_j L^j - \lambda L^{-1} \sum_{j=0}^{\infty} \psi_j \lambda^j]}{1-\lambda L^{-1}}$$

By Equation (A.4) we get

$$\gamma(L) = \frac{\psi(L) - \lambda L^{-1}\psi(\lambda)}{1-\lambda L^{-1}} \tag{A.7}$$

Substituting Equation (A.7) into Equation (A.6) we get

$$\sum_{i=0}^{\infty} \lambda^i E_tX_{t+i} = [\frac{\psi(L) - L^{-1}\lambda\psi(\lambda)}{1-\lambda L^{-1}}]U_t^x \tag{A.8}$$

Noting that $U_t^x = \delta(L)X_t = \psi(L)^{-1}X_t$, rewrite Equation (A.8) as:

$$\sum_{i=0}^{\infty} \lambda^i E_t X_{t+i} = [\frac{I-L^{-1}\lambda\delta(\lambda)^{-1}.\delta(L)}{1-\lambda L^{-1}}]X_t$$

By long division of $\frac{\delta(L)}{1-\lambda L^{-1}}$, we find that

$$\frac{I-L^{-1}\lambda\delta(\lambda)^{-1}.\ (L)}{1-\lambda L^{-1}} = \delta(\lambda)^{-1}[I+\sum_{i=1}^{r-1}(\sum_{i=j+1}^{r} \lambda^{i-j}\delta_i)L^j]$$

Therefore,

$$\sum_{i=0}^{\infty} \lambda^i E_t X_{t+i} = \ell\delta(\lambda)^{-1}[I+\sum_{j=1}^{r-1}(\sum_{i=j+1}^{r} \lambda^{i-j}\delta_i)L^j]P_t \qquad (A.9)$$

Using similar arguments, we have

$$\sum_{i=0}^{\infty} \lambda^i E_t a_{1t+i} = \rho(\lambda)^{-1}[1+\sum_{j=1}^{q-1}\sum_{i=j+1}^{q} \lambda^{i-j}\rho_i)L^j]a_{1t} \qquad (A.10)$$

Equations (A.9) and (A.10) are the optimal prediction formulas.

Note that

$$\lambda^{-1}\sum_{i=0}^{\infty} \lambda^i X_{t+i} = \lambda^{-1}X_t + X_{t+1} + \lambda X_{t+2} + \ldots \qquad (A.11)$$

and

$$\lambda^{-1}[\sum_{i=0}^{\infty} \lambda^i X_{t+i} - X_t] = \sum_{i=0}^{\infty} \lambda^i X_{t+1+i} \qquad (A.12)$$

Equation (A.12) can be used to obtain $\sum_{i=0}^{\infty} \lambda^i E_t X_{t+1+i}$ from Equation (A.9)

APPENDIX B: PLOTS OF CORN ACREAGE (AC) AND SOYBEAN ACREAGE (AS)

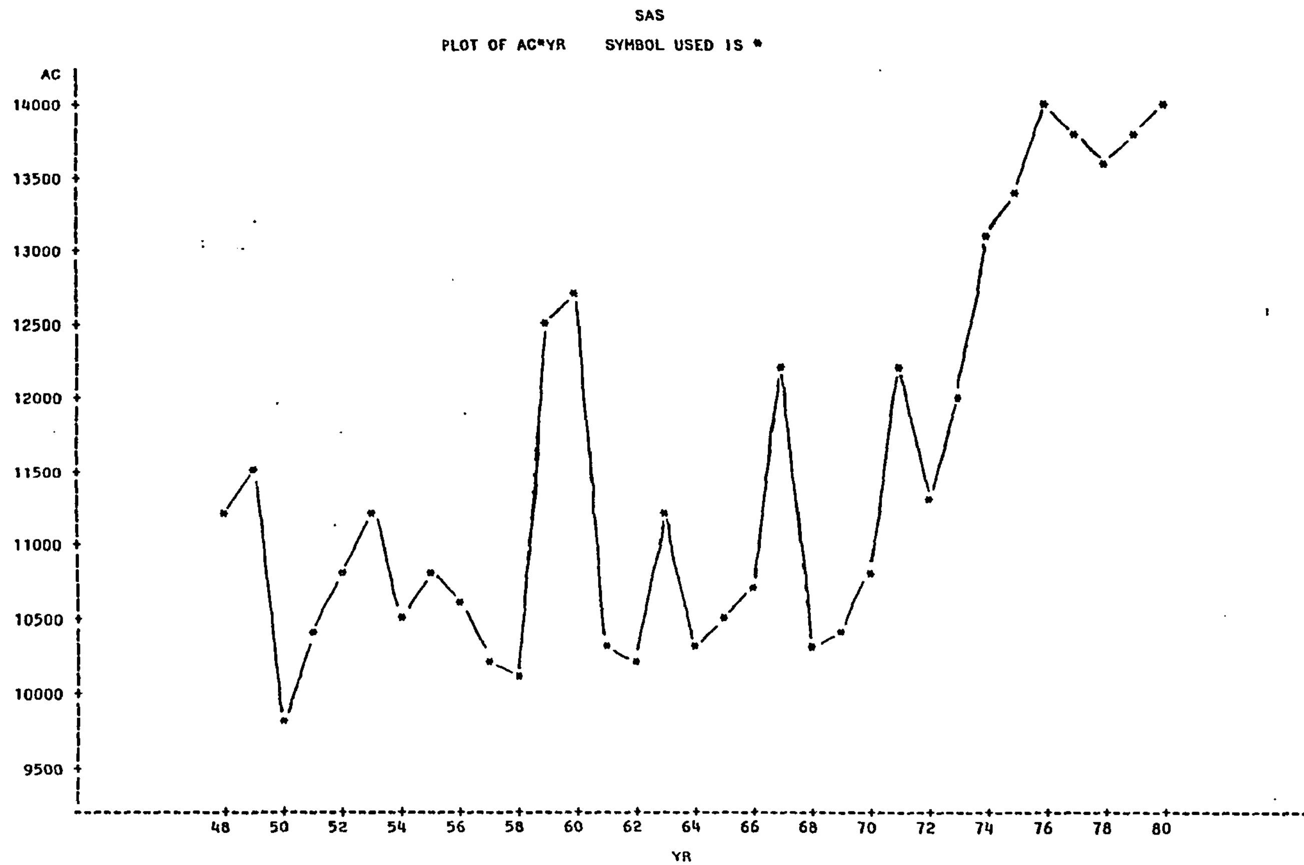

Figure B.1. Plot of area of corn from 1948-1980

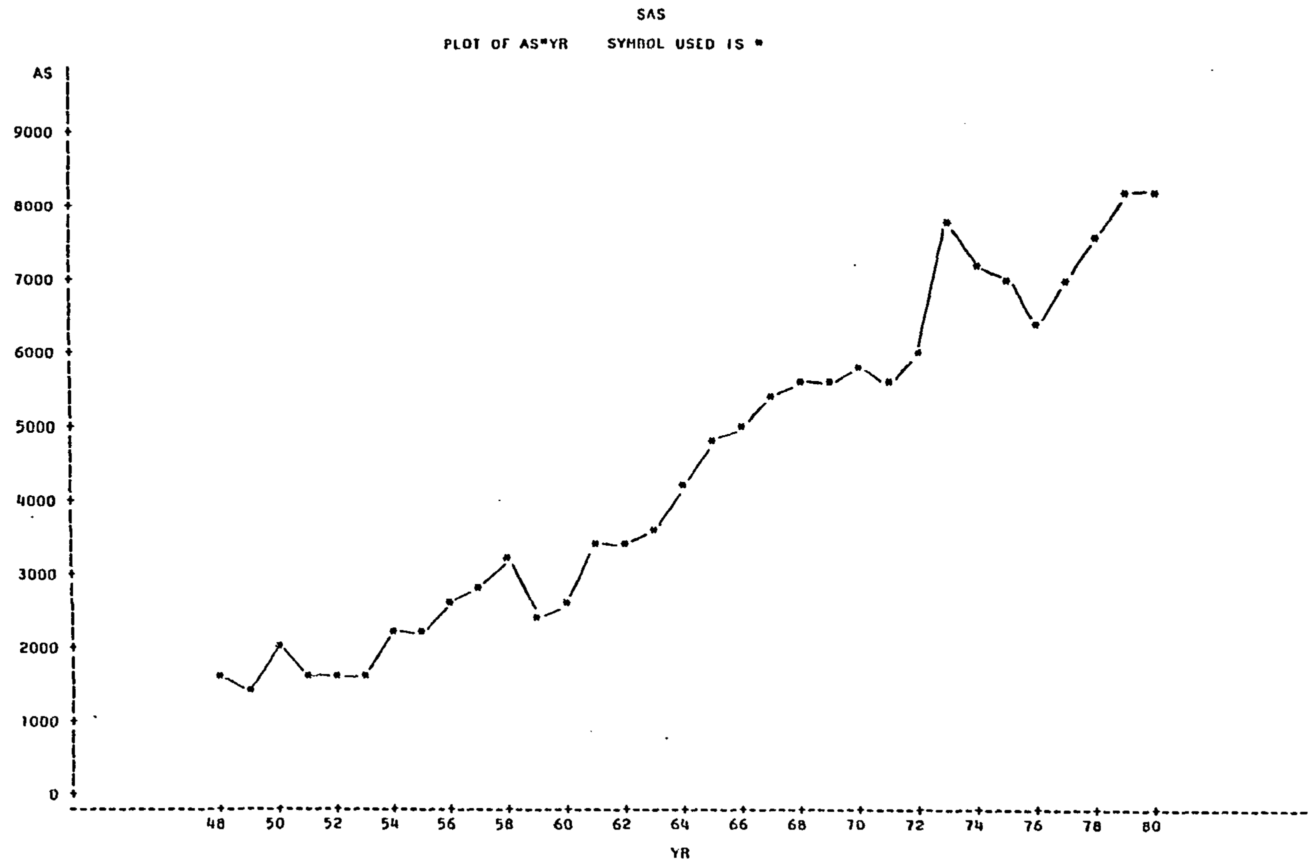

Figure B.2. Plot of area of soybeans from 1948-1980

APPENDIX C: TESTING LAG LENGTH FOR AUTOREGRESSIVE SYSTEM

Lags	T	T-K	$(T-K)\,\text{Log}\left(\frac{\lvert Dr \rvert}{\lvert Du \rvert}\right)$
1 vs. 2	31	21	38.624
2 vs. 3	30	15	40.483
3 vs. 4	29	9	44.876
4 vs. 5	28	3	19.78

$\chi^2(q)$ = 24.3366 (50% level)

= 29.3389 (25% level)

= 34.3816 (10% level)

= 37.6525 (5% level)

= 44.3141 (1% level)

q = 25 is the number of restrictions

APPENDIX D: IMPULSE RESPONSES

Table D.1. Impulse responses to (triangularized) shock in prices K periods after shock

OBS	K	AC	AS	YC	YS	P
1	0	-0.013	0.026	0.060	-0.01	0.700
2	1	-54.600	57.800	5.100	1.60	21.700
3	2	24.300	6.100	-0.970	-1.30	6.300
4	3	7.200	-7.600	-1.030	0.35	1.400
5	4	-2.000	-2.300	0.040	-0.26	4.300
6	5	-2.900	1.800	0.250	0.29	2.000
7	6	1.500	-0.110	-0.050	-0.20	0.900
8	7	-0.370	0.070	0.001	0.12	-0.008
9	8	-0.040	-0.020	0.010	0.08	0.006

Table D.2. Impulse responses to (triangularized) shocks in YS K periods after shock

OBS	K	AC	AS	YC	YS	P
1	0	-0.0007	-0.007	-0.05000	0.030	0.000
2	1	-1.4000	2.500	1.90000	4.000	0.002
3	2	-0.7200	1.500	0.66000	2.800	-0.003
4	3	1.9000	-0.950	0.13000	-0.240	0.001
5	4	-0.9500	0.150	0.30000	0.160	-0.009
6	5	0.4000	0.150	-0.01000	0.007	6.000
7	7	0.2900	-0.120	-0.00100	-0.005	0.000
8	8	-0.1900	0.070	0.00008	0.003	0.000
9	9	0.1200	-0.050	0.00000	-0.002	0.000
10	10	-0.0900	0.040	0.00000	0.002	0.000
11	11	0.0600	-0.020	0.00000	-0.001	0.000
12	12	-0.0400	0.010	0.00000	0.000	0.000

Table D.3. Impulse responses to (triangularized) shocks in YC K periods after shock

OBS	K	AC	AS	YC	YS	P
1	0	-0.0006	0.0005	0.0700	0.000	0.000
2	1	1.9000	-1.8000	2.1000	-0.300	-0.200
3	2	0.4500	-1.0800	-0.5000	0.200	0.230
4	3	-1.5000	0.7900	0.0100	-0.120	-0.010
5	4	0.7900	-0.1500	-0.0100	0.080	0.007
6	5	-0.3400	0.1100	0.0800	-0.060	-0.005
7	6	0.2800	-0.1400	-0.0100	0.040	0.004
8	7	-0.2300	0.0900	0.0001	-0.030	-0.002
9	8	0.1500	-0.0500	0.0000	0.002	0.001
10	9	-0.1000	0.0400	0.0000	-0.001	-0.001
11	10	0.0700	-0.0300	0.0000	0.000	0.000
12	11	-0.0500	0.0200	0.0000	0.000	0.000
13	12	0.0100	-0.0100	0.0000	0.000	0.000

Table D.4. Impulse responses to (triangularized) shocks in AC K periods after shock

OBS	K	AC	AS	YC	YS	P
1	0	0.004	0.00	0.06	0.00	0.00
2	1	25.400	34.70	-3.80	-2.50	5.40
3	2	-7.300	8.03	1.40	0.73	-1.40
4	3	9.400	-6.40	-6.80	-0.78	4.60
5	4	-3.300	-2.10	0.68	0.51	1.70
6	5	0.270	0.79	-0.52	0.30	2.20
7	6	-0.990	1.80	0.64	0.23	-1.70
8	7	1.040	-1.50	-0.58	-0.16	1.08
9	8	-0.590	1.20	0.23	0.11	-0.73
10	9	0.360	0.40	-0.14	0.07	0.53
11	10	-0.270	-0.25	0.13	0.05	-0.38
12	11	0.020	-0.01	-0.09	-0.31	0.26
13	12	0.000	0.00	0.06	0.27	-0.18

Table D.5. Impulse responses to (triangularized) shocks in AS K periods after shock

OBS	K	AC	AS	YC	YS	P
1	0.0009	0.0024	0.00	0.00	0.00	1.00
2	2.0000	8.1000	37.10	-0.71	-0.63	-1.90
3	3.0000	3.1000	29.60	-1.70	-0.67	2.20
4	4.0000	20.9000	5.90	0.16	0.11	-2.30
5	5.0000	2.3000	3.40	0.19	-0.49	0.81
6	6.0000	1.2000	0.95	-0.14	0.26	-0.25
7	7.0000	1.1000	-1.10	0.10	-0.22	0.18
8	8.0000	-1.2000	0.41	0.69	0.17	-0.10
9	9.0000	5.5000	-0.13	-0.17	-0.11	0.07
10	10.0000	0.2100	0.15	0.01	0.07	-0.05
11	11.0000	0.1300	-0.12	-0.01	-0.05	0.03

Table D.6. Percentage of forecast error variance K-years ahead produced by each triangularized innovation

Forecast error in	Innovation in:					
	K	AC	AS	YC	YS	P
AC	0	8.5	1.1	.2	.26	90
	1	21	4.1	2.3	3.6	59
	5	15.9	11.1	13.4	13.9	56.7
	12	28	10	36	15	10
AS	0	0	.78	.04	6.7	92.4
	1	16.6	21.4	2.7	6.3	53.
	5	18.1	34.3	6.9	13.0	27.7
	12	14	24	17	21	19
YC	0	0	0	59.1	30.1	10.8
	1	28.1	5.6	50.1	7	9.1
	5	25.4	5.1	53.1	8.9	7.6
	12	20	5	56	12	6
YS	0	0	0	0	90	10
	1	19	2.5	1.4	66.9	10.2
	5	20.6	3.7	4.1	63.2	9.4
	12	8	3	8	79	1
P	0	0	0	0	0	100
	1	5.1	11.6	.7	.7	82
	5	9.2	13.8	1.6	1.4	74
	12	17	23	3	2	55

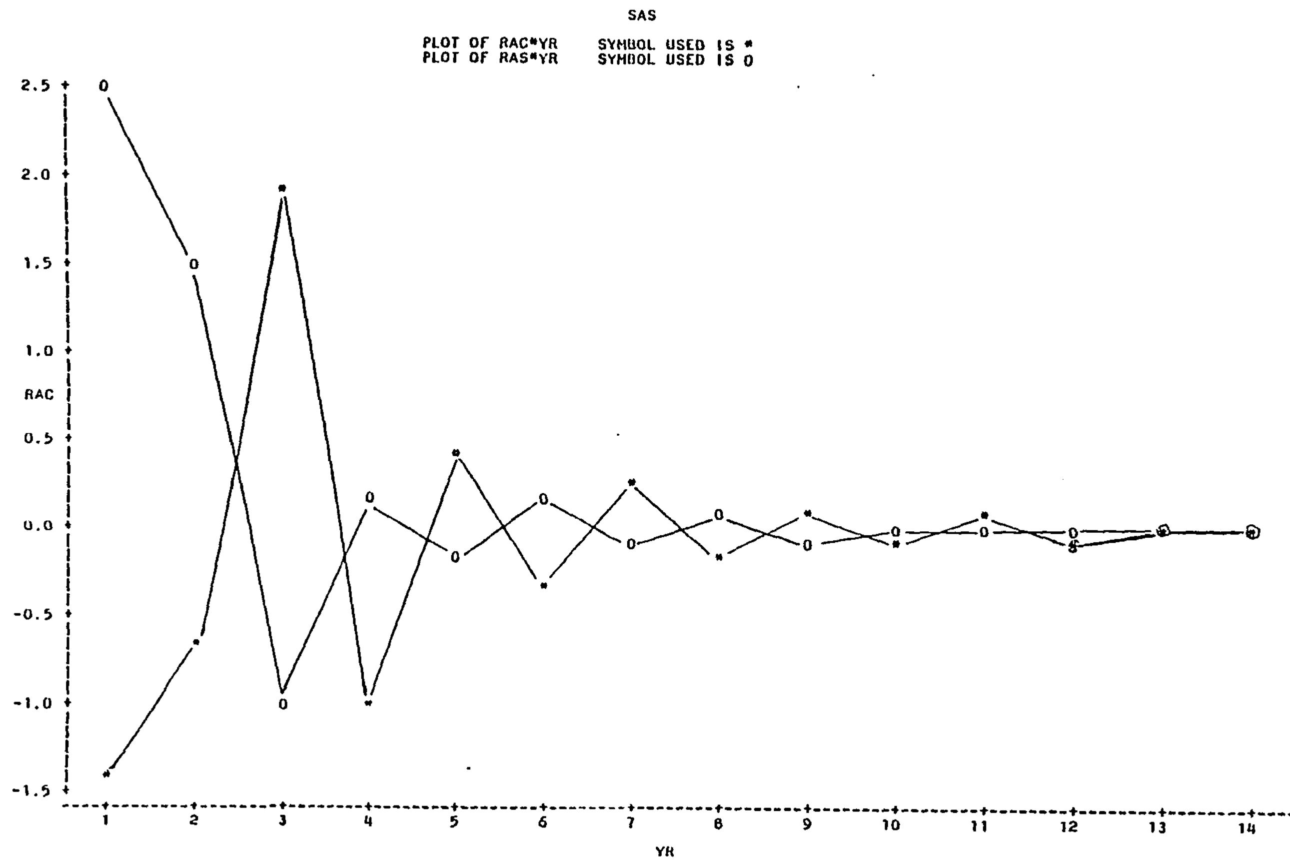

Figure D.1. Plot of responses of area of corn and area of soybeans to one standard deviation shock in yield of soybeans

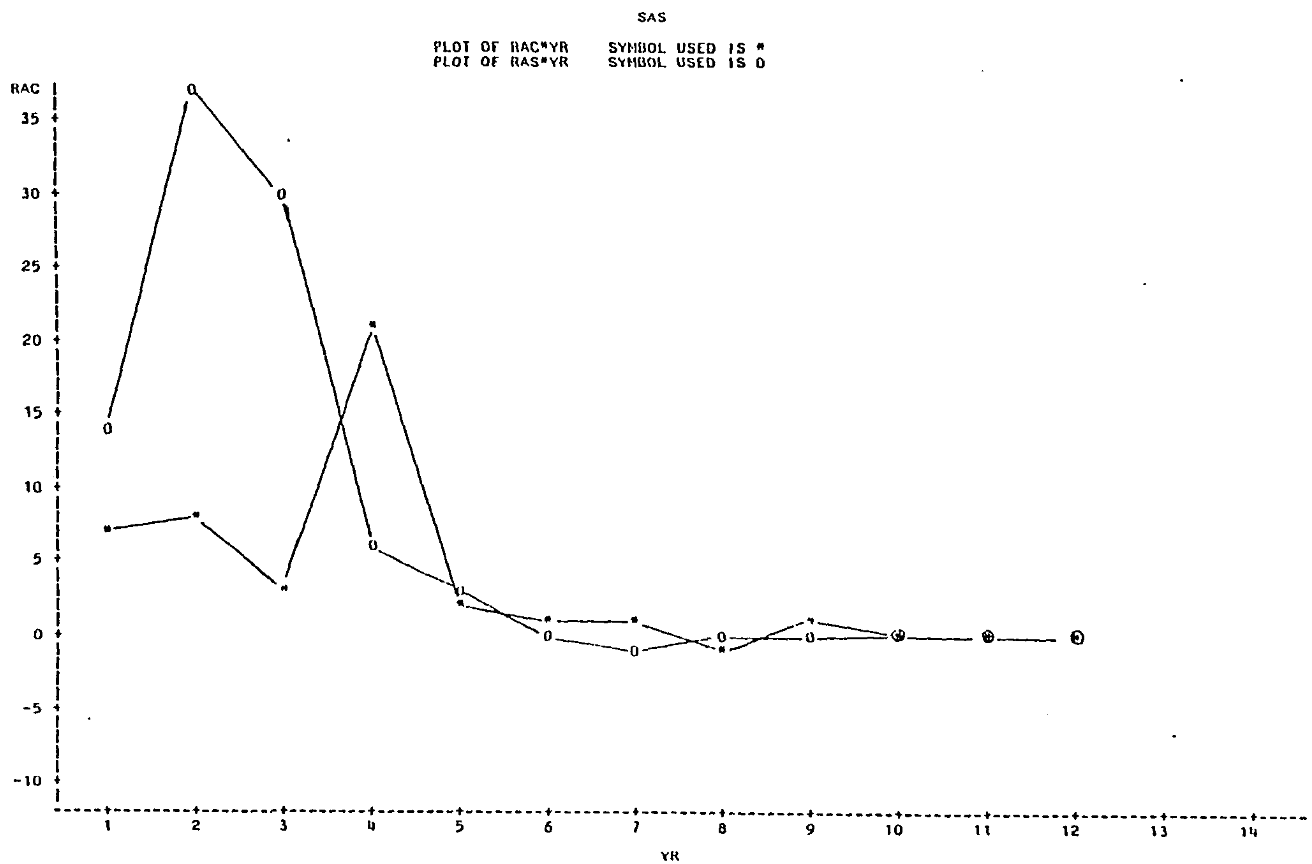

Figure D.2. Plot of responses of area of corn and area of soybeans to one standard deviation shock in area of soybeans

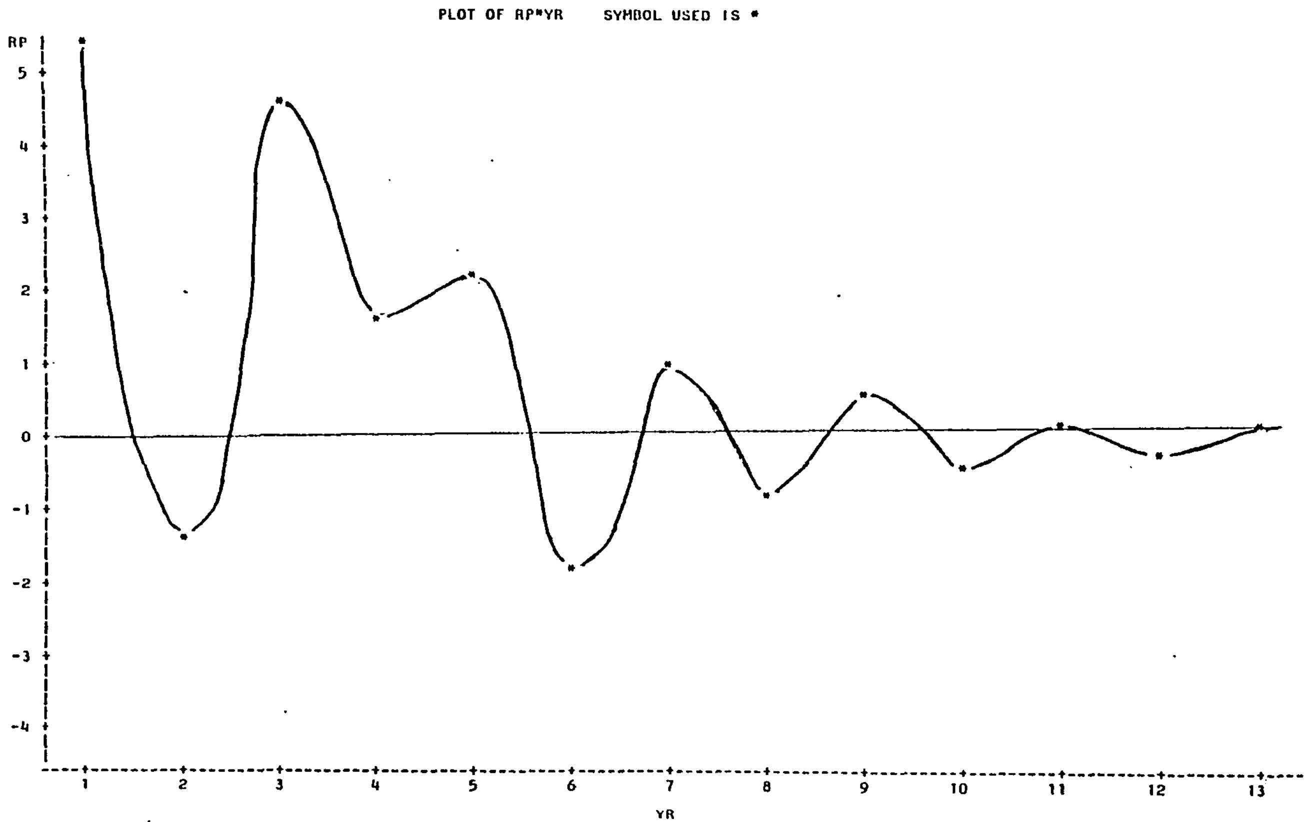

Figure D.3. Plot of responses of price to one standard deviation shock in area of soybeans

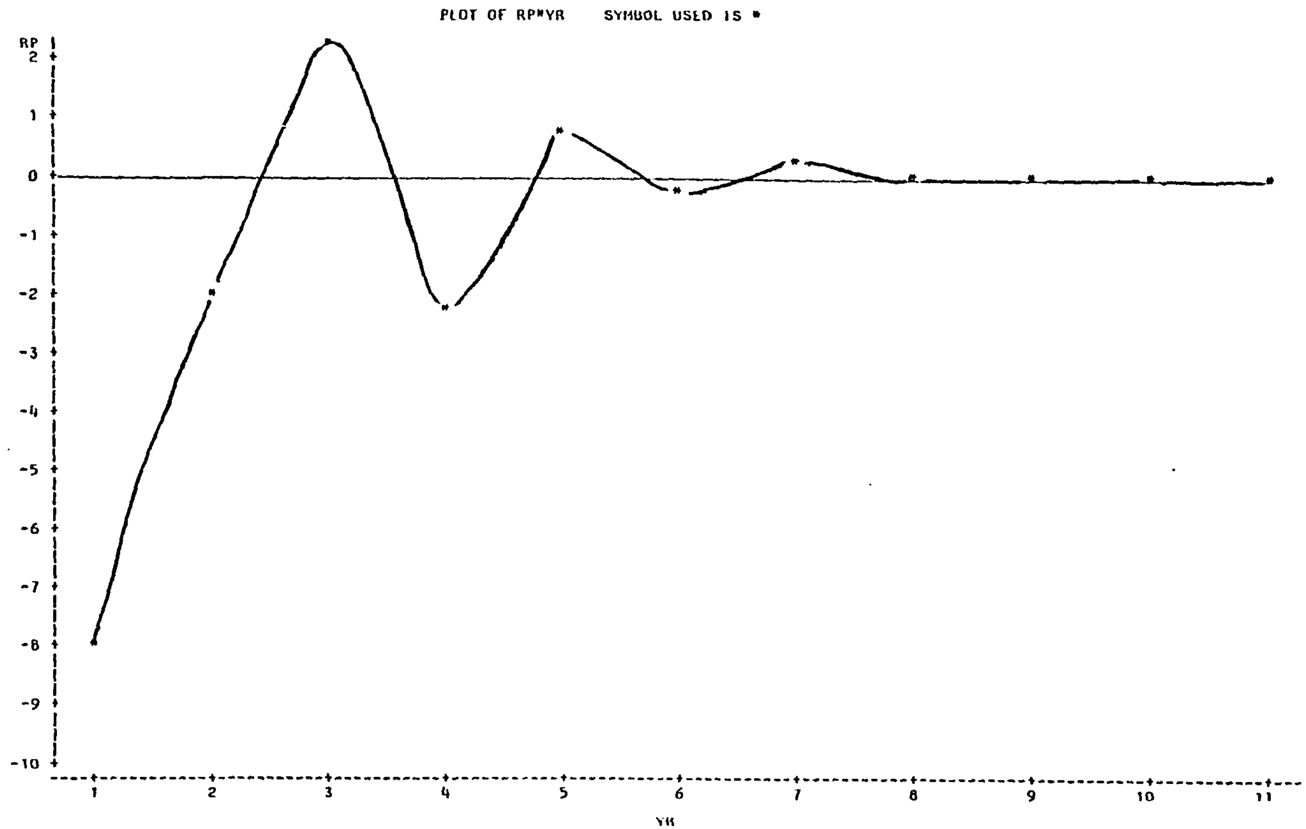

Figure D.4. Plot of response of price to one standard deviation shock in area of corn

APPENDIX E: THE DATA

The data used in this dissertation appear on page . The sample period is from 1948 to 1980. The data were compiled from various publications:

(i) Agricultural Year books, USDA

(ii) Iowa Crop and Livestock Reporting Services, Iowa Department of Agriculture

(iii) Statistical Annuals, Chicago Board of Trade.

Variables:

The first entry in the data set (OBS) is the observation number. The second entry (YR) is the year; 48 stands for 1948 and 80 for 1980. The remaining variables are defined as follows:

AS = area (acreage) of soybeans in thousands of acres

YS = yield of soybeans in bushels per acre

AC = area (acreage) of corn in thousands of acres

YC = yield of corn in bushels per acre

PS = price of soybeans in cents per bushel

PC = price of corn in cents per bushel

PDS = production (output) of soybeans in thousands of bushels

P = relative price of soybeans to price of corn defined as

$$P = \frac{PS}{PC} \cdot \frac{PDS}{AS}$$

GPC = Government support price for corn in cents per bushel

FPK = Futures price of corn (April contract for December delivery) in cents per bushel

FPB = Futures price of corn (April contract for December delivery) in cents per bushel

AT = total acreage cultivated in thousands of acres

Table D.7. The data set

OBS	YR	AS	YS	AC	YC	PS	PC	PDS	P	GPC	FPK	FPB	AT
1	48	1616	22.5	11213	60.5	222	131	35190	36.903	144	171.50	293.00	22144
2	49	1380	23.0	11493	48.0	222	127	30820	39.039	140	116.13	201.75	22208
3	50	1960	22.0	9837	48.5	264	152	42460	37.626	147	127.05	212.00	22548
4	51	1638	20.5	10386	45.0	268	161	32452	32.979	157	171.28	299.50	22336
5	52	1540	25.5	10782	55.0	269	150	38913	45.314	160	171.43	273.50	22336
6	53	1679	21.5	11213	50.0	291	141	35626	43.792	160	156.50	272.38	22336
7	54	2149	26.5	10540	52.0	241	145	56418	43.635	162	144.88	263.13	22444
8	55	2278	20.0	10799	48.5	224	144	45220	30.879	158	137.55	232.38	22685
9	56	2551	20.0	10583	53.0	217	131	50000	32.467	150	143.15	253.63	22685
10	57	2844	27.0	10249	62.0	205	105	76329	52.399	140	128.13	226.25	22620
11	58	3128	25.5	10085	66.0	202	108	79458	47.511	136	119.40	223.03	22620
12	59	2377	26.5	12493	65.0	191	100	62778	50.444	112	115.50	212.65	22873
13	60	2615	25.5	12658	63.5	213	97	66274	55.652	106	110.80	208.88	22208
14	61	3426	28.5	10343	75.5	228	108	97042	59.798	120	117.90	252.00	21861
15	62	3415	27.5	10151	77.1	233	108	93636	59.154	120	117.90	239.53	21514
16	63	3586	30.5	11155	81.5	244	104	109038	71.339	125	114.13	243.28	20473
17	64	4267	28.5	10273	79.0	257	110	121239	66.383	125	119.43	244.13	19952
18	65	4864	26.0	10467	82.0	261	113	126100	59.880	125	121.50	256.70	19722
19	66	5010	29.5	10676	89.0	270	117	147382	67.887	130	120.88	279.05	19665
20	67	5361	27.5	12171	88.5	250	101	144265	66.609	135	139.00	278.25	19320
21	68	5576	32.0	10346	93.0	244	107	177952	72.776	135	123.40	264.50	19320
22	69	5632	33.0	10449	98.0	236	111	179850	67.895	135	118.78	235.38	19287
23	70	5709	32.5	10760	92.0	282	125	184600	72.948	135	120.88	265.25	19952
24	71	5516	32.5	12208	102.0	300	105	178750	92.588	135	142.40	284.20	20640
25	72	6050	36.0	11255	110.0	474	165	216000	102.563	141	128.88	321.15	21266
26	73	7800	34.0	11970	107.0	565	258	26350	7.398	164	156.38	430.15	22295
27	74	7200	28.0	13100	90.0	636	297	199080	59.210	138	253.38	542.50	23085
28	75	7000	34.0	13350	95.0	509	250	236980	68.927	138	264.38	555.50	23256
29	76	6470	31.0	13950	97.0	705	205	199950	106.280	157	264.63	503.50	23598
30	77	7100	35.5	13800	96.0	592	199	251340	105.311	200	264.75	731.00	23940
31	78	7600	37.5	13600	115.0	664	217	283125	113.992	210	252.25	618.25	24265
32	79	8200	37.5	13750	127.0	617	242	306375	95.260	220	267.25	705.38	24140
33	80	8300	39.0	14000	110.0	739	297	322530	96.690	235	296.50	650.25	24140